STRESSES IN SOIL

Stresses at a point in a soil layer are caused by:

1- Self weight of the soil layers (Geostatic Stresses)

2- Added load (Such as buildings, bridges, dams, etc.)

Stresses at a point in a soil mass are divided into two main types:

I- Geostatic Stresses ------ Due to the self weight of the soil mass.

II- Excess Stresses ------ From structures
Distributed Loads

- **Strip Loads (L/B > 9)**
  - Wall Footings
  - Embankments

- **Circular Loads (R)**
  - Storage Tanks

- **Rectangular Loads (BxL)**
  - Spread Footings
  - Mat Foundations
I. Geostatic stresses

I.A. Vertical Stress

Vertical geostatic stresses increase with depth,

There are 3 types of geostatic stresses:

a. Total Stress, $\sigma_{total}$
b. Effective Stress, $\sigma'$
c. Pore Water Pressure, $u$

Total Stress = Effective stress + Pore Water Pressure

$$\sigma_{total} = \sigma' + u$$

I.B. Horizontal Stress or Lateral Stress

$$\sigma_h = K_o \sigma'_v$$

$K_o$ = Lateral Earth Pressure Coefficient

• For normally consolidated soils:
  $$K_o = 1 - \sin\phi'$$

• For over consolidated soils:
  $$K_o = (1 - \sin\phi')(OCR)^{\sin\phi}$$
II. Stress Distribution in Soil Mass:

When applying a load on a half space medium the excess stresses in the soil will decrease with depth.

Like in the geostatic stresses, there are vertical and lateral excess stresses.

1. Vertical Stress Due to a Point Load
   Boussinesq’s solution for elastic behaviour

Boussinesq (1885) solved the problem of stress distribution at any point (X) in a:
- semi-infinite half space (infinite in depth) of
- Homogenous (same soil properties with depth),
- isotropic (same soil properties in all directions) and
- elastic (fully recoverably strains) material

as a result of a point load (Q) applied on the surface:
Point Load Stresses

\[
\Delta \sigma_z = \frac{3Q}{2\pi z^2} \left( \frac{1}{1 + (r/z)^2} \right)^{5/2} = \frac{3Qz^3}{2\pi R^5} = \frac{Q}{z^2} I
\]

where \( I \) is an influence factor, and

\[
I = \frac{3}{2\pi} \left( \frac{1}{1 + (r/z)^2} \right)^{5/2}
\]

\[
\Delta \sigma_\theta = \frac{-Q}{2\pi} \left( 1 - 2\nu \left( \frac{z}{r^2 + z^2} \right)^{1/2} - \frac{1}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right)
\]

\[
\Delta \sigma_r = \frac{Q}{2\pi} \left( \frac{3r^2z}{(r^2 + z^2)^{5/2}} - \frac{z}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right)
\]

\( \nu \) is the Poisson’s ratio
Boussinesq's solution

where

\( Q \) = surface point load
\( z \) = depth of the point \( X \) below \( Q \)
\( r \) = the horizontal distance of point \( X \) from \( Q \)

\( I_p \) = point load influence factor for vertical stress change

(available in standard tables or charts)

\( I_p = \text{Influence factor for the point load} \)

\( \text{Knowing } r/z \text{ ----- } I_p \text{ can be obtained from tables} \)

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2. Vertical Stress Under Corner of a Rectangular Area Carrying Uniform Pressure

The vertical stress at a depth \( z \) below the corner of a rectangular area subject to uniform pressure is

\[
\Delta \sigma_z \text{ (or } \Delta \sigma_v) = q \cdot I_R \text{ or } q \cdot I_p
\]

\[
I_R = \frac{1}{4\pi} \left[ \frac{2mn\sqrt{m^2+n^2}+1}{m^2+n^2+m^2n^2+1} \left( \frac{m^2+n^2+2}{m^2+n^2+1} \right) + \tan^{-1} \left( \frac{2mn\sqrt{m^2+n^2}+1}{m^2+n^2+m^2n^2+1} \right) \right]
\]

\[\Delta \sigma_v = q \cdot I_p\]

where

- \( q \) is the bearing pressure and
- \( I_p \) is the influence factor,
- \( B \): width of the loaded area,
- \( L \): Length of the loaded area.
FIGURE 8
Influence Value for Vertical Stress beneath a Corner of a Uniformly Loaded Rectangular Area

FIGURE 9
Determination of Stress Below Corner of Uniformly Loaded Rectangular Area
4. For a Circular Loaded Area

The excess vertical stress (beneath centre)

\[ \Delta \sigma = q \left[1 - \frac{1}{\left(\frac{r}{z}\right)^2 + 1}^{3/2}\right] \]

- \( q \) is the uniformly distributed pressure on the circular area.

\[ \Delta \sigma \text{ or } \Delta \sigma z = q \cdot I \]

- \( q \) = surface contact pressure
- \( z \) = depth
- \( r \) = radius of uniformly loaded area.
- \( x \) = horizontal distance from the center of the circular area.

[Image of influence value for vertical stress under uniformly loaded circular area (Fourier's Case)]
5. The 2:1-Method

An approximate, but very simple way of looking at the vertical stress distribution with depth.

It assumes that the influence of the load area spreads at 2:1 (1 horizontal to 2 vertical) and that the same pressure is then distributed over the larger area - the so called 2:1 method.
The approximate method is reasonably accurate (compared with Boussinesq’s elastic solution) when $z>B$.

Approximate method for rectangular loads

In preliminary analyses of vertical stress increase under the center of rectangular loads, geotechnical engineers often use an approximate method (sometimes called the 2:1 method).

The vertical stress increase under the center of the load is

$$\Delta\sigma_z = \frac{q_s BL}{(B + z)(L + z)}$$