CHAPTER 4
OPEN CHANNEL HYDRAULICS

4.1 Introduction
Open channel flow refers to any flow that occupies a defined channel and has a free surface. Uniform flow has been defined as flow with straight parallel streamlines. For a closed conduit flowing full, this will usually be satisfied if the conduit is straight and its cross section is constant along a portion of its length. For an open channel flow to be uniform, the same conditions must apply to the channel. That is, it must be straight and have a constant cross section. This is not sufficient, however, because the water surface must also be parallel to the channel slope. In simplest form, uniform open channel flow requires that \( \frac{dy}{dx} = 0 \), where \( y \) is the depth and \( x \) is the horizontal direction. This condition will be considered further in the following section. It should be apparent from the above that uniform flow will rarely occur in the natural channel.

Unsteady flow refers to conditions that change with time at a specific point or section. For example, if at any specified point in the flow the velocity is changing with time, the flow is unsteady. If at all points in the flow the velocity is constant with respect to time, the flow is steady. This is in contrast to non-uniform flow, in which the velocity changes from point to point in the flow direction. It is almost impossible to keep the water surface from changing with time in an unsteady open channel flow. Thus, an unsteady, uniform flow is most unlikely.

We will define the depth in an open channel flow as the vertical distance from the water surface to the lowest point in the channel bed. In a rectangular channel the depth will be constant across the entire section, while in a natural channel the depth at any section must be measured at the deepest point. In a uniform flow the depth will remain constant from section to section.

Because of the free surface, gravity may be expected to play an important role. Just as a pressure gradient was frequently responsible for flow in a closed conduit, gravity acting through the fluid weight causes the water to flow down a slope. Consequently, the Froude number will usually be the significant parameter.

4.2 Classification of free surface flow.
Flow in a channel is characterized by the mean velocity, even though a velocity profile exists at a given section with a relation of

\[
V = \frac{1}{A} \int_A u \, dA
\]  

(4.1)

In the one dimensional model we assume the velocity to be equal to \( V \) everywhere at a given cross section. This model provides excellent results and is used widely. Flows in open channels are most likely to be turbulent, and the velocity profile can be assumed to be approximately constant.

The flow is classified as a combination of steady or unsteady, and uniform or non-uniform. Steady flow signifies that the mean velocity \( V \), as well as the depth \( y \), is independent of time, whereas unsteady flow necessitates that time be considered as an independent variable. Uniform flow implies that \( V \) and \( y \) are independent of the position coordinate in the direction of flow; non-uniform flow signifies that \( V \) and \( y \) vary in magnitude along that coordinate. The possible combinations are shown in Table 4.1
Table 4.1: Combinations of one dimensional free surface flows

<table>
<thead>
<tr>
<th>Type of flow</th>
<th>Average velocity</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady, uniform</td>
<td>V = constant</td>
<td>y = constant</td>
</tr>
<tr>
<td>Steady, non-uniform</td>
<td>V = V(x)</td>
<td>y = y(x)</td>
</tr>
<tr>
<td>Unsteady, uniform</td>
<td>V = V(t)</td>
<td>y = y(t)</td>
</tr>
<tr>
<td>Unsteady, non-uniform</td>
<td>V = V(x,t)</td>
<td>y = y(x,t)</td>
</tr>
</tbody>
</table>

Steady non-uniform flow is a common occurrence in rivers and in man-made channels. In those situations, it will be found to occur in two ways. In relatively short reaches, called transitions, there is a rapid change in depth and velocity; such flow is termed as rapidly varied flow. Examples are the hydraulic jump, flow entering a steep channel from a lake or reservoir, flow close to a free outfall from a channel, and flow in the vicinity of an obstruction such as a bridge pier or a sluice gate.

Along more extensive reaches of channel the velocity and depth may not vary rapidly, but rather, change in a slow manner. Here, the water surface can be considered continuous and the regime is called gradually varied flow. Examples of gradually varied steady flow are the backwater created by a dam placed in a river, and the drawdown of a water surface as flow approaches a falls. The following figure illustrates how both rapidly varied flow (RVF) and gradually varied flow (GVF) can occur simultaneously in a reach of channel.

![Figure 4.1 Steady non-uniform flows in a channel](image)

4.2 Significance of Froude Number

The primary mechanism for sustaining flow in an open channel is gravitational force. The parameter that represents this gravitational effect is the Froude Number.

\[
Fr = \frac{V}{\sqrt{gL}}
\]  

(4.2)

Which is the ratio of inertial force to gravity force. In the context of open channel flow, V is the mean cross sectional velocity, and L is the representative length parameter. For a channel of rectangular cross section, L becomes the depth y of the flow. The Froude number plays the dominant role in open-channel flow analysis. By knowing the magnitude of Froude number one can ascertain significant characteristics regarding the flow regime. For example, if Fr>1, the flow possesses a relatively high velocity and shallow depth; on the other hand, when Fr<1, the velocity is relatively low and the depth is relatively deep. Except in the vicinity of rapids, cascades, and waterfalls, most rivers experience a Froude number less than unity.
4.2 Hydrostatic Pressure Distribution

Consider a channel in which the flow is nearly horizontal as shown in Figure 4.2. In this case there is little or no vertical acceleration of fluid within the reach, and the streamlines remain nearly parallel. Such a condition is common in many open-channel flows, and indeed, if slight variations exist, the streamlines are assumed to behave as if they are parallel. Since the vertical accelerations are nearly zero, one can conclude that in the vertical direction the pressure distribution is hydrostatic. As a result, the sum \( p + \gamma z \) remains a constant at any depth, and the hydraulic grade line coincides with the water surface. It is customary in open channel flow to designate \( z \) as the elevation of the channel bottom, and \( y \) as the depth of flow. Since at the channel bottom \( p/\gamma = y \), the hydraulic grade line is given by the sum \( (y+z) \).

![Figure 4.2 Reach of open channel flow](image)

4.3 Channel Geometry

Channel cross sections can be considered to be either regular or irregular. A regular section is one whose shape does not vary along the length of the channel, whereas, an irregular section will have changes in its geometry.

The simplest channel shape is a rectangular section. The cross-sectional area is given by

\[
A = B_w y
\]

In which \( B_w \) is the width of the channel bottom. Additional parameters of importance for open channel flow are the wetted perimeter, the hydraulic radius, and the width of the free surface. The wetted perimeter \( P \) is the length of the line of contact between the liquid and the channel; for a rectangular channel it is

\[
P = B_w + 2y
\]

The hydraulic radius \( R \) is the area divided by the wetted perimeter, that is,

\[
R = \frac{A}{P} = \frac{B_w y}{B_w + 2y}
\]

The free surface width \( B \) is equal to the bottom width \( B_w \) for a rectangular channel. The cross sectional parameters and necessary formulas for trapezoidal, triangular and circular cross sections are given in Table 4.2:
4.4 Best Hydraulic Section
The conveyance of a channel section increases with an increase in the hydraulic radius or with a decrease in the wetted perimeter. Consequently, the channel section with the smallest wetted perimeter for a given channel section area will have maximum conveyance, referred to as the best hydraulic section or the cross section of greatest hydraulic efficiency. Table 4.3 presents the geometric elements of the best hydraulic sections for six cross section shapes. These cross sections may not always be practical because of difficulties in construction and use of material. Even though the best hydraulic section gives the minimum area for a given discharge, it may not necessarily have the minimum excavation.

### Table 4.2: Geometric Functions for Channel Elements (Chow, 1959)

<table>
<thead>
<tr>
<th>Section</th>
<th>Area $A$</th>
<th>Wetted perimeter $P$</th>
<th>Hydraulic radius $R$</th>
<th>Top width $T$</th>
<th>Hydraulic depth $D$</th>
<th>Section factor $Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$B_0z'$</td>
<td>$(B_0 + z\eta)$</td>
<td>$z\eta^2$</td>
<td>$\frac{1}{8}(\theta - \sin \theta \tan^2 \phi)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trapezoid</td>
<td>$B_0 + 2y$</td>
<td>$B_0 + 2y\sqrt{1 + z^2}$</td>
<td>$2y\sqrt{1 + z^2}$</td>
<td>$\frac{1}{2}x_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangle</td>
<td>$B_0 + 2y$</td>
<td>$(B_0 + z\eta)B_0 + 2y\sqrt{1 + z^2}$</td>
<td>$z\eta$</td>
<td>$\frac{1}{4}(1 - \sin \theta \tan \phi) \eta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>$B_0 + 2y$</td>
<td>$(B_0 + z\eta)B_0 + 2y\sqrt{1 + z^2}$</td>
<td>$z\eta$</td>
<td>$\sin \frac{\theta}{2} \eta$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \frac{2\delta R}{3\delta t} \frac{1}{A \delta z} \left( \frac{5B_0 + 6y}{3y(B_0 + 2y)} \right) = \frac{(B_0 + 2y)(5B_0 + 6y\sqrt{1 - z^2} - 4z\eta\sqrt{1 + z^2})}{3y(y - z\eta + 3y\sqrt{1 + z^2})} \]

\[ \frac{8}{3} \]

\[ \frac{4(2 \sin \theta + 30 - 50 \cos \theta)}{3d_0(6 - \sin \theta \sin (8/2))} \]

where $\theta = 2 \cos^{-1}(1 - 2y)\delta_y$.

### Table 4.3: Best hydraulic sections (Chow, 1959)

<table>
<thead>
<tr>
<th>Cross-section</th>
<th>Area $A$</th>
<th>Wetted perimeter $P$</th>
<th>Hydraulic radius $R$</th>
<th>Top width $T$</th>
<th>Hydraulic depth $D$</th>
<th>Section factor $Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trapezoid, half of a hexagon</td>
<td>$\sqrt{3}y^2$</td>
<td>$2\sqrt{3}y$</td>
<td>$\frac{1}{3}y$</td>
<td>$\frac{4}{3}\sqrt{3}y$</td>
<td>$\frac{3}{2}y^{2.5}$</td>
<td></td>
</tr>
<tr>
<td>Rectangle, half of a square</td>
<td>$2y^2$</td>
<td>$4y$</td>
<td>$\frac{1}{3}y$</td>
<td>$2y$</td>
<td>$y$</td>
<td>$2y^{2.5}$</td>
</tr>
<tr>
<td>Triangle, half of a square</td>
<td>$y^2$</td>
<td>$2\sqrt{2}y$</td>
<td>$\frac{1}{4}\sqrt{2}y$</td>
<td>$2y$</td>
<td>$\frac{1}{3}y$</td>
<td>$\sqrt{2}y^{2.5}$</td>
</tr>
<tr>
<td>Semicircle</td>
<td>$\frac{\pi}{2}y^2$</td>
<td>$\pi y$</td>
<td>$\frac{1}{3}y$</td>
<td>$2y$</td>
<td>$\frac{\pi}{2}y$</td>
<td>$\frac{\pi}{4}y^{2.5}$</td>
</tr>
<tr>
<td>Parabola, $T = 2\sqrt{2}y$</td>
<td>$\frac{1}{3}y \sqrt{2}y^2$</td>
<td>$\frac{1}{3}y \sqrt{2}y^2$</td>
<td>$\frac{1}{3}y$</td>
<td>$2\sqrt{2}y$</td>
<td>$\frac{1}{3}y$</td>
<td>$9y \sqrt{3}y^{2.5}$</td>
</tr>
<tr>
<td>Hydrostatic catenary</td>
<td>$1.39586y^2$</td>
<td>$2.9836y$</td>
<td>$0.46784y$</td>
<td>$1.917532y$</td>
<td>$0.72795y$</td>
<td>$1.19093y^{2.5}$</td>
</tr>
</tbody>
</table>

4.5 Energy concept in steady uniform flow
The energy equation for open channel flow can be derived in a similar manner as the energy equation for pipe flow using the control volume approach. Considering open-channel flow in the
control volume in Figure 4.3, the energy equation can be expressed as
\[
\frac{P_1}{\gamma} + \alpha_1 \frac{v_1^2}{2g} + z_1 - h_L = \frac{P_2}{\gamma} + \alpha_2 \frac{v_2^2}{2g} + z_2
\]  
(4.6)

Where headloss, \( h_L \), is due to viscous stress. The kinetic energy correction factor, \( \alpha \), is used when using the energy principle. The velocity head in open channels is generally greater than the value computed according to the energy equation, thus a correction factor is required. The value of \( \alpha \) for flow in circular pipes is normally ranges from 1.03 to 1.06 for turbulent flow. Because \( \alpha \) is not known precisely, it is not commonly used in pipe flow calculations or accepted as unity. The value of \( \alpha \) for open channel flow varies by the type of channel flow. For example, in regular channels, flumes, and spillways \( \alpha \) range between 1.10 and 1.20 and for river valleys and areas it ranges between 1.5 and 2.0 with an average of 1.75. Note that the value of \( \alpha \) will be considered as 1.0 in our lecture unless a value is attained for special cases.

In steady uniform open channel flows the pressure is hydrostatically distributed, and thus \((P/\gamma+z)\) is constant at each section in the control volume, so that \(P_1/\gamma = y_1\) and \(P_2/\gamma = y_2\). The energy equation for non-uniform open channel flow is expressed as
\[
y_1 + \alpha_1 \frac{v_1^2}{2g} + z_1 - h_L = y_2 + \alpha_2 \frac{v_2^2}{2g} + z_2
\]  
(4.7)

For uniform flow it is well known that the velocity at any section of the channel is same and the water depth is uniform all along the channel, so:
\[
y_1 = y_2
\]
\[
\frac{v_1^2}{2g} = \frac{v_2^2}{2g}
\]

Therefore,
\[
h_L = z_1 - z_2
\]

By dividing both sides by \( L \), the length of the control volume (channel), the following headloss per unit length of channel, \( S_f \) is obtained as
\[
S_f = \frac{h_L}{L} = \frac{z_1 - z_2}{L}
\]  
(4.8)

so that the friction slope equals the channel bottom slope. The channel bottom slope \( S_0 = \tan \theta \), where \( \theta \) is the angle of inclination of the channel. If \( \theta \) is small (<10 degrees), then:
\[
\tan \theta \approx \sin \theta = \frac{h_L}{L} = \frac{z_1 - z_2}{L}
\]  
(4.9)

### 4.6 Momentum concept in steady uniform flow

The forces acting upon the fluid control volume in Figure 4.3 are friction, gravity, and hydrostatic pressure. The friction force, \( F_f \), is the product of the wall shear stress \( \tau_o \) and the area over which it acts, \( PL \), where \( P \) is wetted perimeter of the cross-section, thus
\[
F_f = -\tau_o PL
\]  
(4.10)

In which negative sign indicates that the friction force acts opposite to the direction of flow. The gravity force \( F_g \) relates to the weight of the fluid \( \gamma AL \), where \( \gamma \) is the specific weight of the fluid (weight per unit volume). The gravity force on the fluid is the component of the weight acting in the direction of flow, that is
The hydrostatic forces as shown in the Figure 4.3 are denoted as $F_1$ and $F_2$, and are identical for uniform flow so that $F_1 - F_2 = 0$. For a steady uniform flow, the general form of the integral momentum equation in the x-direction is written as;

$$\sum F = \sum_{cs} v_x (\rho V.A)$$

in which

$$\sum F = F_1 + F_g + F_f - F_2$$

and

$$\sum_{cs} v_x (\rho V.A) = 0$$

Since $F_1 = F_2$ then the above equations result in

$$F_g + F_f = 0$$

or

$$\gamma AL \sin \theta - \tau_o PL = 0$$

for small values of $\theta$,

$$S_o \approx \sin \theta$$

which results in

$$\gamma ALS_o = \tau_o PL$$
for steady uniform flow the friction and gravity forces are in balance and \( S_o = S_f \). Solving the above equation for the wall shear stress yields

\[
\frac{\gamma ALS_o}{PL} = \tau_o
\]

or

\[
\tau_o = \gamma RS_o = \gamma RS_f
\]

where \( R = A/P \) is the hydraulic radius.

The shear stress \( \tau_o \) for fully turbulent flow can be expressed as a function of density, velocity, and resistance coefficient \( C_f \) as

\[
\tau_o = C_f \rho \left( \frac{v^2}{2} \right)
\]

Equating the two shear stress formulas results in

\[
\gamma RS_o = C_f \rho \left( \frac{v^2}{2} \right)
\]

and solving for the velocity gives

\[
v = \sqrt{\frac{2g}{C_f} RS_o}
\]

defining

\[
C = \sqrt{\frac{2g}{C_f}}
\]

then the equation can be simplified to the well-known Chezy Equation

\[
v = C \sqrt{RS_o} \quad (4.11)
\]

where \( C \) is referred to as the Chezy coefficient. Robert Manning (1891, 1895) derived the following empirical relation for \( C \) based upon experiments:

\[
C = \frac{1}{n} R^{1/6}
\]

where \( n \) is the Manning’s roughness coefficient. Substituting \( C \) from the above equation into the velocity equation results in the Manning Equation.

\[
v = \frac{1}{n} R^{2/3} S_o^{1/2} \quad (4.12)
\]

which is valid for the cases \( S_o = S_f \). Values of \( n \) are listed in the Table 4.4

<table>
<thead>
<tr>
<th>Table 4.4: Values of the roughness coefficient ( n ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of channel and description</td>
</tr>
<tr>
<td>A. Closed conduits flowing partly full</td>
</tr>
<tr>
<td>A-1. Metal</td>
</tr>
<tr>
<td>a. Brass, smooth</td>
</tr>
<tr>
<td>b. Steel</td>
</tr>
<tr>
<td>1. Lockbar and welded</td>
</tr>
<tr>
<td>2. Riveted and spiral</td>
</tr>
<tr>
<td>c. Cast iron</td>
</tr>
<tr>
<td>1. Coated</td>
</tr>
<tr>
<td>2. Uncoated</td>
</tr>
<tr>
<td>Type of channel and description</td>
</tr>
<tr>
<td>---------------------------------</td>
</tr>
<tr>
<td>d. Wrought iron</td>
</tr>
<tr>
<td>1. Black</td>
</tr>
<tr>
<td>2. Galvanized</td>
</tr>
<tr>
<td>e. Corrugated metal</td>
</tr>
<tr>
<td>1. Subdrain</td>
</tr>
<tr>
<td>2. Storm drain</td>
</tr>
<tr>
<td>A-2. Nonmetal</td>
</tr>
<tr>
<td>a. Lucite</td>
</tr>
<tr>
<td>b. Glass</td>
</tr>
<tr>
<td>c. Cement</td>
</tr>
<tr>
<td>1. Neat, surface</td>
</tr>
<tr>
<td>2. Mortar</td>
</tr>
<tr>
<td>d. Concrete</td>
</tr>
<tr>
<td>1. Culvert, straight and free of debris</td>
</tr>
<tr>
<td>2. Culvert with bends, connections, and some debris</td>
</tr>
<tr>
<td>3. Finished</td>
</tr>
<tr>
<td>4. Sewer with manholes, inlet, etc., straight</td>
</tr>
<tr>
<td>5. Unfinished, steel form</td>
</tr>
<tr>
<td>6. Unfinished, smooth wood form</td>
</tr>
<tr>
<td>7. Unfinished, rough wood form</td>
</tr>
<tr>
<td>e. Wood</td>
</tr>
<tr>
<td>1. Stave</td>
</tr>
<tr>
<td>2. Laminated, treated</td>
</tr>
<tr>
<td>f. Clay</td>
</tr>
<tr>
<td>1. Common drainage tile</td>
</tr>
<tr>
<td>2. Vitrified sewer</td>
</tr>
<tr>
<td>3. Vitrified sewer with manholes, inlet, etc.</td>
</tr>
<tr>
<td>4. Vitrified subdrain with open joint</td>
</tr>
<tr>
<td>g. Brickwork</td>
</tr>
<tr>
<td>1. Glazed</td>
</tr>
<tr>
<td>2. Lined with cement mortar</td>
</tr>
<tr>
<td>h. Sanitary sewers coated with sewage slimes, with bends and connections</td>
</tr>
<tr>
<td>i. Paved invert, sewer, smooth bottom</td>
</tr>
<tr>
<td>j. Rubble masonry, cemented</td>
</tr>
<tr>
<td>B. Lined or built-up channels</td>
</tr>
<tr>
<td>B-1. Metal</td>
</tr>
<tr>
<td>a. Smooth steel surface</td>
</tr>
<tr>
<td>1. Unpainted</td>
</tr>
<tr>
<td>2. Painted</td>
</tr>
<tr>
<td>b. Corrugated</td>
</tr>
<tr>
<td>B-2. Nonmetal</td>
</tr>
<tr>
<td>a. Cement</td>
</tr>
<tr>
<td>1. Neat, surface</td>
</tr>
<tr>
<td>2. Mortar</td>
</tr>
<tr>
<td>b. Wood</td>
</tr>
<tr>
<td>1. Planed, untreated</td>
</tr>
<tr>
<td>2. Planed, creosoted</td>
</tr>
<tr>
<td>3. Unplaned</td>
</tr>
<tr>
<td>4. Plank with battens</td>
</tr>
<tr>
<td>5. Lined with roofing paper</td>
</tr>
<tr>
<td>Type of channel and description</td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td><strong>c. Concrete</strong></td>
</tr>
<tr>
<td>1. Trowel finish</td>
</tr>
<tr>
<td>2. Float finish</td>
</tr>
<tr>
<td>3. Finished, with gravel on bottom</td>
</tr>
<tr>
<td>4. Unfinished</td>
</tr>
<tr>
<td>5. Gunite, good section</td>
</tr>
<tr>
<td>6. Gunite, wavy section</td>
</tr>
<tr>
<td>7. On good excavated rock</td>
</tr>
<tr>
<td>8. On irregular excavated rock</td>
</tr>
<tr>
<td><strong>d. Concrete bottom float finished with sides of</strong></td>
</tr>
<tr>
<td>1. Dressed stone in mortar</td>
</tr>
<tr>
<td>2. Random stone in mortar</td>
</tr>
<tr>
<td>3. Cement rubble masonry, plastered</td>
</tr>
<tr>
<td>4. Cement rubble masonry</td>
</tr>
<tr>
<td>5. Dry rubble or riprap</td>
</tr>
<tr>
<td><strong>e. Gravel bottom with sides of</strong></td>
</tr>
<tr>
<td>1. Formed concrete</td>
</tr>
<tr>
<td>2. Random stone in mortar</td>
</tr>
<tr>
<td>3. Dry rubble or riprap</td>
</tr>
<tr>
<td><strong>f. Brick</strong></td>
</tr>
<tr>
<td>1. Glazed</td>
</tr>
<tr>
<td>2. In cement mortar</td>
</tr>
<tr>
<td><strong>g. Masonry</strong></td>
</tr>
<tr>
<td>1. Cemented rubble</td>
</tr>
<tr>
<td>2. Dry rubble</td>
</tr>
<tr>
<td><strong>h. Dressed ashlar</strong></td>
</tr>
<tr>
<td>1. Smooth</td>
</tr>
<tr>
<td>2. Rough</td>
</tr>
<tr>
<td><strong>j. Vegetal lining</strong></td>
</tr>
<tr>
<td>C. Excavated or dredged</td>
</tr>
<tr>
<td><strong>a. Earth, straight and uniform</strong></td>
</tr>
<tr>
<td>1. Clean, recently completed</td>
</tr>
<tr>
<td>2. Clean, after weathering</td>
</tr>
<tr>
<td>3. Gravel, uniform section, clean</td>
</tr>
<tr>
<td>4. With short grass, few weeds</td>
</tr>
<tr>
<td><strong>b. Earth, winding and sluggish</strong></td>
</tr>
<tr>
<td>1. No vegetation</td>
</tr>
<tr>
<td>2. Grass, some weeds</td>
</tr>
<tr>
<td>3. Dense weeds or aquatic plants in deep channels</td>
</tr>
<tr>
<td>4. Earth bottom and rubble sides</td>
</tr>
<tr>
<td>5. Stony bottom and weedy banks</td>
</tr>
<tr>
<td>6. Cobble bottom and clean sides</td>
</tr>
<tr>
<td><strong>c. Dragline-excavated or dredged</strong></td>
</tr>
<tr>
<td>1. No vegetation</td>
</tr>
<tr>
<td>2. Light brush on banks</td>
</tr>
<tr>
<td><strong>d. Rock cuts</strong></td>
</tr>
<tr>
<td>1. Smooth and uniform</td>
</tr>
<tr>
<td>2. Jagged and irregular</td>
</tr>
<tr>
<td><strong>c. Channels not maintained, weeds and brush uncut</strong></td>
</tr>
<tr>
<td>1. Dense weeds, high as flow depth</td>
</tr>
<tr>
<td>2. Clean bottom, brush on sides</td>
</tr>
<tr>
<td>3. Same, highest stage of flow</td>
</tr>
<tr>
<td>4. Dense brush, high stage</td>
</tr>
</tbody>
</table>
Manning’s Equation in SI units for uniform flow can also be expressed as

\[ Q = \frac{A}{n} R^{2/3} S^{1/2} \]  

(4.13)

We can use the manning’s formula for discharge to calculate steady uniform flow. Two calculations are usually performed to solve uniform flow problems.

a) Discharge from a given depth

b) Depth for a given discharge

In steady uniform flow the flow depth is generally known as normal depth.
Question 4-1
A concrete lined trapezoidal channel with uniform flow has a normal depth of 2-m. The base width is 5-m and the side slopes are equal at 1:2. Manning’s $n$ can be taken as 0.015 and the bed slope $S=0.001$. Calculate the discharge and the mean velocity in the open channel.

Solution 4-1
The first step is to calculate the section properties of the open channel

$$A = (5 + 2y)y = 18m^2$$

$$P = 5 + 2y\sqrt{1 + 2^2} = 13.94m$$

Using the Manning’s Equation

$$Q = \frac{A}{n}R^{2/3}S_y^{1/2} = \frac{18}{0.015}\left(\frac{18}{13.94}\right)^{2/3}0.001^{1/2} = 45 m^3/sec$$

The simplest way to calculate the mean velocity is to use the continuity equation which yields:

$$V = \frac{Q}{A} = 45 \div 18 = 2.5 m/sec$$

Question 4-2
Using the same channel as above which is a concrete lined trapezoidal channel with uniform flow has a normal depth of 2-m. The base width is 5-m and the side slopes are equal at 1:2. Manning’s $n$ can be taken as 0.015 and the bed slope $S=0.001$. Calculate the normal depth if the discharge in channel is known to be 30$m^3$/sec.

Solution 4-2
The first step is to calculate the section properties of the open channel

$$A = (5 + 2y)y = 18m^2$$

$$P = 5 + 2y\sqrt{1 + 2^2} = 13.94m$$

Using the Manning’s Equation

$$30 = \frac{A}{n}R^{2/3}S_y^{1/2} = \frac{(5 + 2y)y}{0.015}\left(\frac{(5 + 2y)y}{5 + 2y\sqrt{1 + 2^2}}\right)^{2/3}0.001^{1/2}$$

$$30 = 2.108(5 + 2y)y\left(\frac{(5 + 2y)y}{5 + 2y\sqrt{1 + 2^2}}\right)^{2/3}$$

We need to calculate $y$ from this equation. Even for this quite small geometry the equation we need to solve for normal depth is complex. One simple strategy to solve this is to select some
appropriate values of \( y \) and calculate the right hand side of this equation and compare it to \( Q(=30) \) in the left. When it equals to \( Q \) it means that we have the correct \( y \). Even though there will be several solutions to this equation, this strategy generally works. In this case from the previous example we know that at \( Q=45\text{-m}^3/\text{sec} \), \( y=2\text{-m} \). So at \( Q=30\text{-m}^3/\text{sec} \) then \( y<2\text{-m} \).

Guess that \( y=1.7\text{-m} \) then \( Q=32.7\text{-m}^3/\text{sec} \)

Guess that \( y=1.6\text{-m} \) then \( Q=29.1\text{-m}^3/\text{sec} \)

Guess that \( y=1.63\text{-m} \) then \( Q=30.1\text{-m}^3/\text{sec} \)

**Question 4-3**

Using the same channel as above the channel is developed into the following section, called compound channel section, in order to withstand for the flood conditions. When the flow goes over the top of the trapezoidal channel it moves to the flood plains so the section allows for a lot more discharge to be carried. If the flood channels are 10-m wide and have side slopes of 1:3 and the manning roughness coefficient \( n=0.035 \). Find the discharge for a flood level of 4-m.

**Solution 4-3**

A channel section which is composed of different roughness along its wetted perimeter is called a *composite section*. A channel section, in which the cross section is composed of several distinct subsections are called *compound section*.

The conveyance \( (K) \) is a measure of the discharge carrying capacity of a channel, defined by the equation,

\[
Q = KS_o^{1/2} \tag{4.14}
\]

For any given water depth its value may be found by equating (4.14) with Manning equation given in Equation (4.14). Therefore, the conveyance is given as

\[
K = \frac{A^{5/3}}{nP^{2/3}} \tag{4.15}
\]

For the solution of the above question it is necessary to split the channel cross-section into subsections (1), (2) and (3) as shown in the above figure. Mannings formula may be applied to each one in turn, and the discharges can be summed. The division of the section into subsections is a little arbitrary. If the shear stress across the arbitrary divisions is small compared with the bed shear stresses, it may be ignored to obtain an approximate solution.

For section (1)
\[ A_1 = \left( \frac{5 + 15}{2} \right) 2.5 + (15 \times 1.5) = 47.5m^2 \]

and

\[ P_1 = 5 + 2(\sqrt{5^2 + 2.5^2}) = 16.18m \]

Using the Manning’s Equation

\[ K_1 = \frac{A^{5/3}}{nP^{2/3}} = \frac{47.5^{5/3}}{0.015 \times 16.18^{2/3}} = 6492.5 \]

Sections (2) and (3) have the same dimensions, hence

\[ A_2 = A_3 = \left( \frac{10 + 14.5}{2} \right) 1.5 = 18.38m^2 \]

and

\[ P_2 = P_3 = 10 + (\sqrt{4.5^2 + 1.5^2}) = 14.74m \]

Using the Manning’s Equation

\[ K_2 = K_3 = \frac{A^{5/3}}{nP^{2/3}} = \frac{18.38^{5/3}}{0.035 \times 14.74^{2/3}} = 608.4 \]

Hence,

\[ Q_1 = KS_1^{1/2} = \frac{A^{5/3}}{nP^{2/3}} S_1^{1/2} = 6492.5 \times 0.001^{1/2} = 205.3m^3/\text{sec} \]

\[ Q_2 = Q_3 = KS_3^{1/2} = \frac{A^{5/3}}{nP^{2/3}} S_3^{1/2} = 608.4 \times 0.001^{1/2} = 19.2m^3/\text{sec} \]

Therefore, the total discharge of flow in the compound channel is

\[ Q = Q_1 + Q_2 + Q_3 = 205.3 + 19.2 + 19.2 = 243.7m^3/\text{sec} \]

**Question 4-4**

Consider a trapezoidal channel cross-section as shown in the following figure. The manning roughness coefficient values for each side of channel is different from each other, whereas \(n_1=0.02\); \(n_2=0.018\) and \(n_3=0.028\). Determine the discharge in the channel if the channel base width is 5m, \(z_1=2\), \(z_2=1.5\) and the normal depth is 2 meters. The slope of the channel is 0.004.
A channel section which is composed of different roughness along its wetted perimeter is called a composite channel. In such sections an equivalent manning roughness coefficient is used to define a unique channel cross-section. Horton in 1933 has derived a formula for equivalent roughness which can be written as:

\[
\left( \frac{1}{n_e} \right)^{2/3} = \left( \frac{b}{h} \right)^{2/3} \left( \frac{\sqrt{z_1^2 + \ln n_1^{3/2}} + \sqrt{z_2^2 + \ln n_2^{3/2}}} {P} \right)^{2/3} + y + bn_2^{3/2} 
\]  

In which the discharge of flow in the channel can be written as

\[
Q = \frac{A^{5/3}}{n_e P^{2/3}} S_o^{1/2} 
\]  

According to the above definition

\[
A = \left( \frac{3 \times 2}{2} \right) + \left( \frac{4 \times 2}{2} \right) + (5 \times 2) = 17 m^2 
\]  

and

\[
P = 5 + (\sqrt{3^2 + 2^2}) + (\sqrt{4^2 + 2^2}) = 13.08 m 
\]  

Using the equivalent Manning’s roughness formula

\[
n_e = \left( \frac{\sqrt{2^2 + 1(0.02)^{3/2}} + \sqrt{1.5^2 + 1(0.018)^{3/2}}}{13.08} \right)^{2/3} + 5(0.028)^{3/2} 
\]  

Inserting all these into manning’s formula results

\[
Q = \frac{17^{5/3}}{(0.0227)(13.08)^{2/3}} \sqrt{0.004} = 56.4 m^3 / sec 
\]  

4.7 Specific Energy Concept in Open Channel Flow

The fundamental energy equation, developed states that losses will occur for a real fluid between any two sections of the channel, and hence the total energy will not remain constant. The energy balance is given simply by the relation

\[
H_1 - h_i = H_2 
\]  

The energy at any location along the channel is the sum of the vertical distance measured from a horizontal datum \( z \), the depth of flow \( y \), and the kinetic energy \( v^2/2g \). That sum defines the energy grade line and is termed the total energy, \( H \).

\[
H = z + y + \frac{v^2}{2g} 
\]  

The kinetic energy correction factor associated with the \( v^2/2g \) term is assumed to be unity. This
is common practice for most prismatic channels of simple geometry since the velocity profiles are nearly uniform for the turbulent flows involved. Furthermore, accepting that the pressure distribution in open channel flow is hydrostatic (i.e., $p=\gamma y$) and using the channel bottom as the datum (i.e., $z=0$), then total head above the channel bottom can be defined as *Specific Energy, $E$.  

$$E = y + \frac{v^2}{2g} \quad (4.20)$$

Using continuity equation ($Q=Av$), the specific energy can be expressed in terms of the discharge as

$$E = y + \frac{Q^2}{2gA^2} \quad (4.21)$$

Specific Energy Curves, such as shown in Figures 4.3 and 4.4 can be derived using Specific energy equation expressed in terms of discharge.

At a constant discharge, minimum specific energy occurs when the critical flow conditions are favorable, $F_r=1$ (i.e., $dE/dy=0$), so that:

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^2} \frac{dA}{dy} = 0$$
Referring to Figure 4.3, the top width is defined as $T=dA/dy$ so the above equation can be expressed as:

$$1 - \frac{TQ^2}{gA} = 0$$

or

$$\frac{TQ^2}{gA^2} = 1$$

to denote the critical conditions one can use the subscript “c” to define $T$, $A$, $v$ and $y$ as $T_c$, $A_c$, $v_c$ and $y_c$, so

$$\frac{T_cQ^2}{gA_c^2} = 1$$

or

$$v_c^2 = \frac{A_c}{T_c}$$

Rearranging the above equation yields,

$$\frac{v_c^2}{g\left(\frac{A_c}{T_c}\right)} = 1$$

The *hydraulic depth* is defined as $D=A/T$ so the final form of the equation becomes

$$\frac{v_c^2}{g(D_c)} = 1$$

or

$$\frac{v_c}{\sqrt{gD_c}} = 1$$

This is basically the *Froude Number* $F_r$, which is 1 at critical flow:

- subcritical flow: $F_r = \frac{V}{\sqrt{gD}} < 1$
- critical flow: $F_r = \frac{V}{\sqrt{gD}} = 1$
- supercritical flow: $F_r = \frac{V}{\sqrt{gD}} > 1$

Figure 4.4 illustrates the range of subcritical flow and the range of supercritical flow along with the location of the critical states. Note the relationship of the specific energy curves and the fact that $Q_3 > Q_2 > Q_1$. Figure 4.3 illustrates the alternate depths $y_1$ and $y_2$ for which $E_1 = E_2$ or

$$y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g}$$

For a rectangular channel $D_c = A_c/T_c = y_c$, so the Froude number for critical flow becomes

$$\frac{v_c}{\sqrt{gy_c}} = 1$$

If we let $q$ be the flow rate per unit width of rectangular channel, then, $q=Q/b$ and $T=b$ yields the
following equation
\[ \frac{T_c Q^2}{g A^3_c} = 1 \]  (4.22)
as
\[ \frac{T_c q^2 b^2}{g T_c^3 y_c^3} = 1 \]
and solving for \( y_{cr} \) will yield:
\[ y_c = \left( \frac{q^2}{g} \right)^{1/3} \]  (4.23)
or in other terms can be written as;
\[ y_c = \left( \frac{Q^2}{b^2 g} \right)^{1/3} \]
Inserting the definition of \( y_{cr} \) into the Energy equation will result in minimum Energy relationship. If
\[ E_{min} = y_{cr} + \frac{Q^2}{2 gy_{cr}^2 b^2} = y_{cr} + \frac{1}{2} y_{cr}^2 + \frac{Q^2}{2 y_{cr} b^2 g} = y_{cr} + \frac{1}{2} y_{cr}^2 y_{cr}^3 = \frac{3}{2} y_{cr} \]
This means that at the critical stage of flow (critical depth) the velocity head equals half the critical hydraulic depth. If \( y=y_{cr} \) and \( Fr=1 \) and the critical stage of flow represents the boundary between supercritical and subcritical flow. If the energy of flow \( E \), is kept constant and \( Q \) depends on \( y \) as follows;
\[ \frac{v^2}{2g} = E - y \]
\[ v = \sqrt{2g(E - y)} \]
Since \( Q=A v \)
\[ Q = by \sqrt{2g(E - y)} \]
When the depth of flow is zero \( y=0 \) the discharge of flow is equivalent to none “0” and when the depth of flow is equivalent to specific Energy, there should be a certain depth at the interval \( 0<y<E \) for which \( Q_{max} \) exists.
\[ \frac{dQ}{dy} = 0 \]
\[ \frac{d^2 Q}{dy^2} < 0 \] \( \rightarrow Q_{max} \)
This gives \( E=1.5y \) when \( Q \) becomes maximum and \( E=1.5y \) is the condition for the critical flow as found above. Therefore, for a given cross section the critical depth yields minimum energy and maximum discharge conditions.
\[ Q_{max} = b y_{cr} \sqrt{2g(E - y_{cr})} \]
\[ Q_{max} = b \frac{2}{3} E \sqrt{2g(E - \frac{2}{3} E)} \]
\[ Q_{\text{max}} = 1.71bE^{3/2} \]

When plotted, specific energy curve \( E = f(y) \) and Koch’s parabola \( Q = f(y) \) are obtained.

![Image](image_url)

**Figure 4.4** Variation of specific energy and specific discharge with depth: (a) \( E \) versus \( y \) for constant \( q \); (b) \( q \) versus \( y \) for a constant \( E \).

**Question 4-5**
Compute the critical depth for a rectangular channel whose bottom width is 4 meters at a discharge of 3m³/sec. The manning roughness coefficient is \( n = 0.02 \). The slope of the channel is 0.004.

**Solution 4-5**
The Froude Number for critical flows is equivalent to 1. Therefore, the magnitude of inertia forces should be equal to the gravitational forces.

\[ V = \sqrt{gD} \]

In which \( D \) is the hydraulic depth that can be expressed as

\[ D = \frac{A}{T} \]

Writing down the cross sectional area, \( A \) and top width, \( T \) in terms of channel geometric parameters results in

\[ D = \frac{by_{cr}}{b} = y_{cr} \]

\[ V = \frac{Q}{A} = \frac{Q}{by_{cr}} = \sqrt{gy_{cr}} \]

\[ V = \frac{Q}{A} = \frac{3}{4 \times y_{cr}} = \sqrt{9.81 \times y_{cr}} \]

\[ 0.75 \frac{y_{cr}}{y_{cr}} = \sqrt{9.81 \times y_{cr}} \]
0.239 = y_{cr}^{3/2}

0.385m = y_{cr}

**Question 4-6**
For a rectangular channel of 20-meters width, construct a family of specific energy curves for Q= 0, 1.36, 2.7, 8.1-m³/sec. Plot the critical points and read minimum energy and critical water depth values. Find also the minimum energy values and critical depths via theoretical calculations.

**Solution 4-6**
The specific energy is computed using Equation (4.21):

\[
E = y + \frac{Q^2}{2gA^2} = y + \frac{Q^2}{2g(20y)^2} = y + \frac{Q^2}{7848y^2}
\]

By the help of the above equation specific energies for different discharges are listed at the table below.

<table>
<thead>
<tr>
<th>Depth y (m)</th>
<th>Q= 0 (m³/sec)</th>
<th>Q= 1.36 (m³/sec)</th>
<th>Q= 2.7 (m³/sec)</th>
<th>Q= 8.1 (m³/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>0.03</td>
<td>0.29</td>
<td>1.06</td>
<td>9.32</td>
</tr>
<tr>
<td>0.04</td>
<td>0.04</td>
<td>0.19</td>
<td>0.62</td>
<td>5.27</td>
</tr>
<tr>
<td>0.05</td>
<td>0.05</td>
<td>0.14</td>
<td>0.42</td>
<td>3.39</td>
</tr>
<tr>
<td>0.06</td>
<td>0.06</td>
<td>0.13</td>
<td>0.32</td>
<td>2.38</td>
</tr>
<tr>
<td>0.07</td>
<td>0.07</td>
<td>0.12</td>
<td>0.26</td>
<td>1.78</td>
</tr>
<tr>
<td>0.08</td>
<td>0.08</td>
<td>0.12</td>
<td>0.23</td>
<td>1.39</td>
</tr>
<tr>
<td>0.09</td>
<td>0.09</td>
<td>0.12</td>
<td>0.20</td>
<td>1.12</td>
</tr>
<tr>
<td>0.1</td>
<td>0.1</td>
<td>0.12</td>
<td>0.19</td>
<td>0.94</td>
</tr>
<tr>
<td>0.2</td>
<td>0.2</td>
<td>0.21</td>
<td>0.22</td>
<td>0.41</td>
</tr>
<tr>
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<td>0.3</td>
<td>0.3</td>
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</tr>
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<td>0.41</td>
<td>0.45</td>
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<td>0.5</td>
<td>0.53</td>
</tr>
<tr>
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<td>0.6</td>
<td>0.6</td>
<td>0.62</td>
</tr>
<tr>
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<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.72</td>
</tr>
<tr>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.81</td>
</tr>
</tbody>
</table>
Computing critical depths for the flow rates using Equation (4.23) results:

\[ Q = 0 \rightarrow y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = 0 \]

\[ Q = 1.36 \rightarrow y_c = \left(\frac{(1.36/20)^2}{9.81}\right)^{\frac{1}{3}} = 0.077m \]

\[ Q = 2.7 \rightarrow y_c = \left(\frac{(2.7/20)^2}{9.81}\right)^{\frac{1}{3}} = 0.123m \]

\[ Q = 8.1 \rightarrow y_c = \left(\frac{(8.1/20)^2}{9.81}\right)^{\frac{1}{3}} = 0.255m \]

4.8 Use of the Energy Equation in Transitions

A transition is a relatively short reach of channel where the depth and velocity change, creating a non-uniform, rapidly varied flow situation. The mechanism for such changes in the flow is usually an alteration of one or more geometric parameters of the channel. Within such regions, the energy equation can be used effectively to analyze the flow in a transition or to aid in the design of a transition.

4.8.1 Channel Constriction

Consider a rectangular channel whose bottom is raised by a distance \( h \) over a short region as shown in Figure 4.5. The change in depth in the transition can be analyzed by use of the energy equation, and as a first approximation one can neglect losses. Assume that the specific energy upstream of the transition is known. Recognizing that \( H=E+z \) energy equation between location 1 and location 2 can be written as

\[ H_1 - h_1 = H_2 \]

Which can be further defined as

\[ E_1 - h = E_2 \]
The depth $y_2$ at the end of the transition can be visualized by inspection of the $E$-$y$ diagram. If the flow at location 1 is subcritical, $y_1$ is located on the upper leg as shown in the specific energy diagram. The magnitude of $h$ is selected to be relatively small such that $y_2 > y_c$, hence the flow at location 2 is similarly subcritical; however, as $h$ is increased still further, a state of minimum energy is ultimately reached in the transition. The condition of minimum energy is sometimes referred to as a choking condition or as choked flow. Once choked flow occurs, as $h$ increases, the variations in depth and velocity are no longer localized in the vicinity of the transition. Influences may be observed for significant distances both upstream and downstream of the transition.

It is important to note that in a transition region, a narrowing of the channel width will create a situation similar to an elevation of the channel bottom. The most general transition region is one that possesses a change in both width and in bottom elevation.

**Question 4-7**

A rectangular channel 3-m wide is conveying water at a depth $y_1=1.55$-m, and velocity $v_1=1.83$-m/sec. The flow enters a transition region as shown in Figure 4.5, in which the bottom elevation is raised by $h=0.20$-m. Determine the depth and velocity in the transition, and the value of $h$ for choking to occur.

**Solution 4-7**

The specific discharge in the channel can be written as

$$\frac{Q}{b} = q = V_1 y_1$$

$$q = 1.83 \times 1.55 = 2.84 \text{ m}^2 / \text{sec}$$

The Froude number at location 1 is

$$F_r = \frac{V_1}{\sqrt{g y_1}} = \frac{1.83}{\sqrt{9.81 \times 1.55}} = 0.47$$

which is less than unity. Hence the flow at location 1 is subcritical. The specific energy at location 1 is found to be;
\[ E_i = y_1 + \frac{v_1^2}{2g} = 1.55 + \frac{1.83^2}{2 \times 9.81} = 1.72m \]

The specific energy at location 2 is found by using the relationship

\[ E_i - h = E_2 \]

\[ E_2 = E_i - h = 1.72 - 0.20 = 1.52m \]

it is important to check if choking occurs or not. Therefore, the critical Energy point must be found.

\[ y_c = \left( \frac{q^2}{g} \right)^{1/3} = \left( \frac{2.84^2}{9.81} \right)^{1/3} = 0.94m \]

As it is proved before critical energy for rectangular channels can be given as

\[ E_c = \frac{3}{2} y_c = 3 \times \frac{0.94}{2} = 1.41m \]

Hence, since \( E_2 > E_c \) we can proceed with calculating \( y_2 \). The depth \( y_2 \) can be evaluated by substituting known values into Energy equation

\[ E_2 = 1.52 = y_2 + \frac{q^2}{2gy_2^2} \]

Which results in

\[ y_2 = 1.26m \]

Therefore,

\[ q = V_2 y_2 \]

\[ V_2 = \frac{q}{y_2} = \frac{2.84}{1.26} = 2.25m/s \]

The flow at location 2 is subcritical since there is no way in which the flow can become supercritical in the transition with the given geometry. The value of \( h \) for critical flow to appear at location 2 is determined by setting

\[ E_i - h = E_c \]

\[ h = E_i - E_c = 1.72 - 1.40 = 0.31m \]

4.9 The hydraulic jump and hydraulic drop theory

Change of the state of flow from subcritical to supercritical or vice versa occurs frequently in open channels. Such change is manifested by a corresponding change in depth of flow from a high stage to a low stage or vice versa. Hydraulic drop is a change in the depth of flow from a high stage to a low stage. More important from a practical viewpoint is the hydraulic jump, a rapid change in the depth of flow from a low stage to a high stage. It occurs frequently in
a channel below the regulation sluice, at the foot of a spillway, or at a place where a steep channel slope suddenly turns flat.

![Figure 4.6 Hydraulic Drop and Hydraulic Jump](image)

A hydraulic jump may occur as shown in Figure 4.6. The uniform flow in the channel to the right of B is given by:

\[ v = \frac{1}{n} R^{2/3} S_o^{1/2} < \sqrt{gy} \]

Because of the hydraulic drop at the left, the flow becomes rapid left of B with its smallest depth at point A. Because of the high velocity between A and B, the slope of the energy grade line is very steep. According to the Specific Energy curve the depth must increase accordingly. There will be a point B, where the steep energy grade line from the left and the flat energy grade line from the right cross each other.

At B,

\[ y_1 + \frac{v_1^2}{2g} = y_2 + \frac{v_2^2}{2g} \]

The energy of the tranquil flow right of point B is equal to the energy of the rapid flow left of point B. The depth now has to change from \( y_1 \) to \( y_2 \). This change can occur only by a jump. The depth cannot follow continuously the specific-energy curve because by passing \( y_{cr} \) steadily the flow would gain energy by increasing depth from \( y_{cr} \) to \( y_2 \). But the energy grade line cannot have a slope against the direction of the flow. Therefore the only way to proceed from \( y_1 \) to \( y_2 \) with a positive slope of the energy grade line is to jump abruptly. But in reality, such a vertical wall of water cannot exist. This front of water will collapse in its upstream direction and form a so-called “surface roller”.

All forms of jumps cause energy losses because of the sudden retardation. Because of the energy losses the jump does not occur under the intersection point of the two energy grade lines but moves upstream to the point which satisfies the condition. As seen in the figure the jump takes place from \( y_1 \) to \( y_2 \).
As a result, the energy loss associated with the jump is considered to be unknown, so the energy equation is not used in the initial analysis. Assuming no friction along the bottom and no submerged obstacle, a dimensionless Froude term can be obtained

\[ Fr_1^2 = \frac{y_2}{2y_1} \left( \frac{y_2}{y_1} + 1 \right) \]  \hspace{1cm} (4.24)

Provided that the Froude number is known, solving the above equation for \( \frac{y_2}{y_1} \) results in

\[ y_2 = \frac{y_1}{2} \left( \sqrt{1 + 8 Fr_1^2} - 1 \right) \]  \hspace{1cm} (4.25)

It is worth to note that it is also valid if the subscript on the depths and Froude number can be reversed.

\[ y_1 = \frac{y_2}{2} \left( \sqrt{1 + 8 Fr_2^2} - 1 \right) \]  \hspace{1cm} (4.26)

The theoretical energy loss associated with a hydraulic jump in a rectangular channel can be determined once the depths and flows at locations 1 and 2 are known. The energy equation is applied from 1 to 2, including the head loss across the jump.

\[ \Delta E = E_1 - E_2 = \left( y_1 + \frac{V_1^2}{2g} \right) - \left( y_2 + \frac{V_2^2}{2g} \right) \]  \hspace{1cm} (4.27)

Substituting \( q = Vy \) results in

\[ \Delta E = y_1 - y_2 + \frac{q^2}{2g} \left( \frac{1}{y_1^2} - \frac{1}{y_2^2} \right) \]  \hspace{1cm} (4.28)

The upstream Froude Number is related to \( q \) as follow:

\[ Fr_1 = \frac{1}{\sqrt{gy_1}} = \frac{\frac{q}{y_1}}{\sqrt{gy_1}} \]

\[ Fr_1^2 = \left( \frac{q}{gy_1} \right)^2 \]  \hspace{1cm} (4.29)

Substituting (4.24) and (4.29) into (4.28), the solution is
\[ \Delta E = \frac{(y_2 - y_1)^3}{4y_2y_1} \]  \hspace{1cm} (4.30)

It should be noted that the energy loss increases very sharply with the relative height of the jump. Table 4.5 shows the various forms that a jump may assume relative to the upstream Froude number. A steady, well-established jump, with \(4.5 < F_r < 9.0\), is often used as an energy dissipator downstream of a dam or spillway.

<table>
<thead>
<tr>
<th>(F_r)</th>
<th>(y_2/y_1)</th>
<th>Classification</th>
<th>Sketch</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;1</td>
<td>1</td>
<td>Jump impossible</td>
<td><img src="image1" alt="Sketch" /></td>
</tr>
<tr>
<td>1 to 1.7</td>
<td>1 to 2.0</td>
<td>Standing wave or undulant jump</td>
<td><img src="image2" alt="Sketch" /></td>
</tr>
<tr>
<td>1.7 to 2.5</td>
<td>2.0 to 3.1</td>
<td>Weak jump</td>
<td><img src="image3" alt="Sketch" /></td>
</tr>
<tr>
<td>2.5 to 4.5</td>
<td>3.1 to 5.9</td>
<td>Oscillating jump</td>
<td><img src="image4" alt="Sketch" /></td>
</tr>
<tr>
<td>4.5 to 9.0</td>
<td>5.9 to 12</td>
<td>Stable, well-balanced steady jump; insensitive to downstream conditions</td>
<td><img src="image5" alt="Sketch" /></td>
</tr>
<tr>
<td>&gt;9.0</td>
<td>&gt;12</td>
<td>Rough, somewhat intermittent strong jump</td>
<td><img src="image6" alt="Sketch" /></td>
</tr>
</tbody>
</table>

It is characterized by the existence of breaking waves and rollers accompanied by a submerged jet with significant turbulence and dissipation of energy in the main body of the jump; downstream, the water surface is relatively smooth. For Froude numbers outside the range 4.5 to 9.0, less desirable jump exist and may create undesirable downstream surface waves. The length of a jump is the distances from the front face to just downstream where smooth water exists; a steady jump has an approximate length of 5.5 times the upstream depth.

**Question 4-8**

A hydraulic jump is situated in a 4-m wide rectangular channel. The discharge in the channel is 7.5 m\(^3\)/sec, and the depth upstream of the jump is 0.2 m. Determine the depth downstream of the jump, the upstream and downstream Froude numbers, and the rate of energy dissipation by the jump.

**Solution 4-8**

Find the unit discharge and upstream Froude number.

\[
\frac{Q}{b} = q = \frac{7.5}{4} = 1.88 \text{ m}^2/\text{sec}
\]

The Froude number at

\[
F_{r1} = \frac{q}{\sqrt{g y_1^3}} = \frac{1.88}{\sqrt{9.81 \times 0.2^3}} = 6.71
\]

The downstream depth is computed, using Equation (4.25), to be

\[
y_2 = \frac{y_1}{2} \left( \sqrt{1 + 8F_{r1}^2} - 1 \right)
\]
The downstream Froude number is
\[
F_{r2} = \frac{q}{\sqrt{gy_2^3}} = \frac{1.88}{\sqrt{9.81 \times 1.8^3}} = 0.25
\]

The head loss in the jump is given by Equation (4.30)
\[
\Delta E = \frac{(y_2 - y_1)^3}{4y_2y_1}
\]
\[
\Delta E = \frac{(1.8 - 0.2)^3}{4 \times 0.2 \times 1.8} = 2.84m
\]

Hence, the rate of energy dissipation in the jump is
\[
\gamma Q \Delta E = 9800 \times 7.5 \times 2.84 = 2.09 \times 10^5 W = 209kW
\]

The length of the jump may be defined as the distance measured from the front face of the surface roller to a point on the surface immediately downstream from the roller. This length cannot be determined easily by theory but has been investigated experimentally by many hydraulicians. In general, it can be said, that the length of the hydraulic jump \( L \) varies between 4.5–6.5\( y_2 \) for Froude number between 4 and 15.

**Question 4-9**

A hydraulic jump occurs in a channel in which the discharge in the channel is 60\( m^3/sec \). The width of the channel is 15-m and the velocity of the flow is 12.5-m/sec at the spillway. The depth \( y_2 \) in the canal after the hydraulic jump is 2.5-m. If Manning’s \( n \) of the concrete lined channel is \( n=0.013 \)

Determine:

a) The energy losses due to friction and due to hydraulic jump
b) The length from toe to the hydraulic jump.
c) The length of the jump
d) What energy is lost in \( H_p \), between \( y \) and \( y_1 \), and what energy is lost in the jump.
Solution 4-9

It is important to find the Specific energy of the flow just at the foot of the spillway, which is simply given by

\[ E = y + \frac{v^2}{2g} \]

The cross sectional area of the rectangular channel is

\[ A = by = \frac{Q}{V} = \frac{60}{12.5} = 15 \times y \]

\[ y = 0.32m \]

Therefore the specific energy is given by,

\[ E = y + \frac{v^2}{2g} = 0.32 + \frac{12.5^2}{2g} = 8.28m \]

Because of the continuity of the flow the discharge in the channel is same everywhere. Using the depth of flow after the hydraulic jump, one can simply find the energy of the flow after the hydraulic jump.

\[ A = by = \frac{Q}{V} = \frac{60}{V_2} = 15 \times 2.5 \]

\[ V_2 = \frac{60}{15 \times 2.5} = 1.6m/sec \]

\[ E_2 = y + \frac{v_2^2}{2g} = 2.5 + \frac{1.6^2}{2g} = 2.63m \]

This means that from the foot of the spillway to the end of the hydraulic jump the total energy loss is

\[ \Delta E = 8.28 - 2.63 = 5.65m \]

We can now use the Equation (4.26) to measure the depth of water just before the hydraulic jump occurs.

\[ y_1 = \frac{v_2}{2} \left( \sqrt{1 + 8Fr_2^2} - 1 \right) \]

\[ y_1 = \frac{2.5}{2} \left( \sqrt{1 + 8 \times \frac{1.6^2}{9.81 \times 2.5}} - 1 \right) = 0.44m \]

and due to the continuity

\[ V_1 = \frac{Q}{A} = \frac{60}{15 \times 0.44} = 9.1m/sec \]

Thus the energy of the flow just before the hydraulic jump is

\[ E_1 = y + \frac{v_1^2}{2g} = 0.44 + \frac{9.1^2}{2g} = 4.66m \]

Now the energy loss due to hydraulic jump can be measured by

\[ \Delta E_{\text{jump}} = E_1 - E_2 = 4.66 - 2.63 = 2.03m \]

and the energy loss before the water reaches to section 1 is

\[ \Delta E_{\text{friction}} = E_1 - E_2 = 8.28 - 4.66 = 3.62m \]
The slope of the energy grade line from the toe of the spillway up to hydraulic jump is

$$ S = \frac{\Delta E}{L} = \frac{3.62}{L} $$

The length of flow from toe of the spillway up to section 1 has two different water depths, 0.32-m and 0.4-m. The average of the two depths can be taken as the average flow depth in this section. Using the manning’s equation one can find the Length up to the hydraulic jump.

$$ Q = \frac{A}{n} R^{2/3} S^{1/2}_o $$

$$ A = by_{avg} = 15 \times \left( \frac{0.32 + 0.4}{2} \right) = 15 \times 0.36 = 5.4m $$

$$ R = \frac{A}{P} = \frac{5.4}{2 \left( \frac{0.32 + 0.4}{2} \right) + 15} = 0.3435 $$

$$ L = 41.65m $$

$$ F_{r1} = \frac{V}{\sqrt{gy}} = \frac{9.1}{\sqrt{9.81 \times 0.44}} = 4.37 $$

It is previously mentioned that the length of jump is around 5.5 times the depth of the downstream part.

$$ L_{jump} = 5.5 \times 2.5 = 13.75m $$

The energy lost between y and y1 is

$$ P = \gamma Q \Delta E_{friction} = 1000 \times 60 \times 3.6 = 216000kgm/s $$

$$ P = 216000 / 75 = 2880Hp $$

The energy lost between y1 and y2 is

$$ P = \gamma Q \Delta E_{jump} = 1000 \times 60 \times 2.03 = 122400kgm/s $$

$$ P = 122400 / 75 = 1630Hp $$

**Question 4-10**

A rectangular channel, 6 m wide, laid on a slope s=0.083, carries 12m³/sec with a mean velocity of 7.5m/sec and discharges onto a 6m wide horizontal apron

a) What is the roughness of the channel?
b) What water elevation above apron is needed to create a hydraulic jump 8 m from the end of the channel?

c) What is the energy lost in the jump?

Solution 4-10

The first step is to find the water depths while approaching to hydraulic jump region

\[ Q = AV = byV \]

\[ 12 = AV = 6 \times y \times 7.5 \]

\[ y = 0.27m \]

Using the manning's equation.

\[ V = \frac{1}{n} R^{2/3} S_n^{1/2} \]

\[ 7.5 = \frac{1}{n} R^{2/3} 0.083^{1/2} \]

\[ A = by = 6 \times 0.27 = 1.62m \]

\[ P = b + 2y = 6 + 2(0.27) = 6.54m \]

\[ R = \frac{A}{P} = \frac{1.62}{6.54} = 0.248 \]

\[ 7.5 = \frac{1}{n} 0.248^{2/3} 0.083^{1/2} \]

\[ n = 0.015 \]

The energy equation in the system can be written as

\[ E - h_L = E_1 \]

\[ y + \frac{v_1^2}{2g} - h_L = y_1 + \frac{v_1^2}{2g} \]

\[ 0.27 + \frac{7.5^2}{2 \times 9.81} - h_L = y_1 + \frac{v_1^2}{2g} \]

\[ 3.14 - h_L = y_1 + \frac{v_1^2}{2g} \]

\[ 3.14 = y_1 + \frac{v_1^2}{2g} + h_L \]

Since the length of the hydraulic jump is 8 meters, then the slope of the flow is

\[ S = \frac{h_L}{L} = \frac{h_L}{8} \]
Therefore rewriting the energy balance equation results in:

\[
\begin{align*}
3.14 &= y_1 + \frac{Q^2}{(6y_1)^2}9.81 + \frac{8Q^2n^2}{(6y_{avg})^2R^{4/3}} \\
3.14 &= y_1 + \frac{12^2}{(6y_1)^2}9.81 + \frac{812^20.015^2}{(6y_{avg})^2R^{4/3}} \\
3.14 &= y_1 + \frac{0.204}{y_1} + \frac{0.0072}{y_{avg}R^{4/3}}
\end{align*}
\]

<table>
<thead>
<tr>
<th>$y$</th>
<th>$y_1$</th>
<th>$(y_1)^2$</th>
<th>$y_{avg}$</th>
<th>$(y_{avg})^2$</th>
<th>$R$</th>
<th>$R^{4/3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27</td>
<td>0.35</td>
<td>0.122</td>
<td>0.31</td>
<td>0.096</td>
<td>0.28</td>
<td>0.182</td>
</tr>
<tr>
<td>0.27</td>
<td>0.33</td>
<td>0.109</td>
<td>0.30</td>
<td>0.09</td>
<td>0.273</td>
<td>0.177</td>
</tr>
<tr>
<td>0.27</td>
<td>0.30</td>
<td>0.09</td>
<td>0.285</td>
<td>0.081</td>
<td>0.26</td>
<td>0.165</td>
</tr>
</tbody>
</table>

Therefore the depth of flow at section 1 is 0.30 meters.

\[
V_1 = \frac{Q}{A} = \frac{12}{6\times0.3} = 6.67m/\text{sec}
\]

Thus the energy of the flow just before the hydraulic jump is

\[
E_1 = y_1 + \frac{v_1^2}{2g} = 0.3 + \frac{6.67^2}{2g} = 2.56m
\]

Using the relationship between the two depths before and after the hydraulic jump results in:

\[
y_2 = \frac{y_1}{2} \left(1 + 8F_{r_1}^2 - 1\right) \\
y_2 = 1.5m \text{ and}
\]

\[
V_2 = \frac{Q}{A} = \frac{12}{6\times1.5} = 1.33m/\text{sec}
\]

\[
E_2 = y_2 + \frac{v_2^2}{2g} = 1.5 + \frac{133^2}{2g} = 1.59m
\]

\[
\Delta E_{jump} = E_1 - E_2 = 2.56 - 1.59 = 0.97m
\]

The energy lost between $y_1$ and $y_2$ is

\[
P = \gamma Q \Delta E_{jump} = 1000 \times 12 \times 0.97 = 11640\text{kgm/s}
\]

\[
P = 11640 / 75 = 155Hp
\]

**Question 4-11**

Water flows at a rate of 15 m³/s in a rectangular channel of 5 m wide. The channel slope is 0.0025 and the Manning coefficient is 0.02.

a) Determine the uniform flow depth and the regime of the flow.
b) The channel width will be reduced by a frictionless sidewall. Determine the minimum width of constricted channel that will not result in a rise in the upstream water surface.

---

**Question 4-12**

A flow of \( q = 3.13 \text{ m}^3/\text{s.m} \) is carried in a rectangular channel at a depth of 1.85 m.

a) Determine the regime of the flow.

b) Prove that the flow is unable to pass through a section, where a frictionless hump of 70 cm high is installed across the bed, with its existing specific energy.

c) Find the change caused by the hump in the upstream water surface elevation.

d) Determine the water depth over the hump.

e) Determine the uniform depth of flow downstream of the hump by taking into consideration the raised upstream water surface elevation.

f) A hydraulic jump occurs at a downstream section distant from the hump. Determine the water depth after this hydraulic jump.

---

**Question 4-13**

In a rectangular channel, the height of a hump is determined so that it will not cause a rise in the upstream water surface. The height of this hump is 0.50 m and the water depth over the hump is 1.50-m. Determine the discharge, the depth(s), and the type(s) of flow.

---

**Question 4-14**

A flow of 1 m³/s is carried in a rectangular channel of 1.3-m wide at a depth of 1-m.

a) Determine the regime of the flow.

b) A frictionless sidewall constriction reduces the channel width to 1 m. Test if the water is able or not to pass through this section with its existing specific energy.

---

**4.10 The gradually varied flow theory**

Open channel flows are characterized primarily by the exposure of a free surface to atmospheric pressure. Therefore, the prime motivating force for open channel flow is due to gravity. Open channels, in general, may be either natural or artificial. Natural channels are uncontrolled and have loose boundaries changing by erosion and deposition. Artificial channels are man-made canals and they have fixed solid boundaries. Natural channels include all water courses that exist naturally in the earth, such as brooks, streams, small and large rivers, and tidal estuaries. Artificial channels are those constructed or developed by human effort, navigation channels, power canals, irrigation canals and flumes, etc.
Flow conditions in open channels are complicated by the fact that the position of free surface is likely to change with respect to time and space. On the other hand, the depth of flow, discharge, and bottom slope of the channel are dependent upon each other.

If the cross-sectional shape, size, and bottom slope of a channel is constant, it is termed as prismatic channel. Generally the man-made channels are included in this group. Nearly all natural channels have irregular cross-sections and consequently are non-prismatic.

Open channel flows may be classified in various ways. A more widely accepted method of classification is made according to the variation of depth of flow with time and space. There are two types of open channel flows, namely laminar and turbulent which are identified by the Reynolds number. Unsteady flows, also called transients, occur in an open channel when the flow properties, such as depth, y, vary with time, t, at a section

\[
\frac{\partial y}{\partial t} \neq 0
\]

As a corollary, if the depth does not change with time, the flow is termed as steady

\[
\frac{\partial y}{\partial t} = 0
\]

If space is taken as criterion, the open channel flow can be either uniform or non-uniform. If the flow properties, such as depth and velocity, remain constant along the length of the channel, the flow is said to be uniform

\[
\frac{\partial y}{\partial x} = 0
\]

whereas a flow in which the flow properties vary along the channel is termed as non-uniform or varied flow

\[
\frac{\partial y}{\partial x} \neq 0
\]

In gradually varied flows, the change of depth is gradual so that the curvature of streamlines is not excessive. Frictional resistance plays an important role in these flows. In rapidly varied flows, curvature of streamlines is large and the depth changes appreciably over short lengths. In other words, rapidly varied flows are local phenomenon and the frictional resistance is relatively insignificant. Details of classification and the methods of solutions are outlined below.
4.10.1 steady non-uniform flow, direct step method
The solution for the differential equation of gradually varied steady non-uniform flow gives the shape of the flow profile:

\[ \frac{dy}{dx} = \frac{S_0 - \overline{S_f}}{1 - \overline{F}_r^2} \]  

(4.31)

where \( S_0 \) is the bed slope, \( \overline{S_f} \) is the average friction slope, and \( \overline{F}_r \) is the average Froude number between two successive sections of the channel.

There are three main methods used for the computations of gradually varied flow: direct integration method, graphical integration method, and numerical step method. A step method is characterized by dividing the channel into short reaches, each being bounded by cross-sections of known hydraulic properties. The computation is principally performed step by step from one end of the reach to the other. The calculations are carried out in the upstream direction if the flow is
subcritical and in the downstream direction otherwise. The upstream and downstream sections are, therefore, the control sections where there exists a relation between the depth and the discharge to simplify the computations.

Direct step method is simple and preferable for prismatic channels, whereas the standard step method is best suited for natural channels.

In general, a step method is characterized by dividing the channel into short reaches and carrying the computation step by step from one end of the reach to the other. The direct step method is a simple step method applicable to prismatic channels.

![Figure 4.8 A channel section](image)

The total head, H, can be given as;

$$H = z + y + \frac{v^2}{2g} \quad (4.32)$$

where, the energy, E, is

$$E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2} \quad (4.33)$$

Taking the derivatives of both sides will result in;

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dE}{dx} = \frac{dz}{dx} + \frac{dE}{dy} \frac{dy}{dx} \quad (4.34)$$

from Figure (4.8) it can be seen that the energy slope, $S_f$, and bed slope $S_0$ can be related with the total head and bed level elevations with respect to the distance interval $\Delta x$.

$$\frac{dH}{dx} = -S_f \quad (4.35)$$

and

$$\frac{dz}{dx} = -S_0 \quad (4.36)$$

Therefore the Equation (4.34) can be written as;

$$S_0 - S_f = \frac{dE}{dy} \frac{dy}{dx} \quad (4.37)$$

The first derivative of the energy E, with respect to y, can be taken solved by considering the Equation (4.33). The mathematical solution of the derivation finally results in,

$$\frac{dE}{dy} = 1 - Fr^2 \quad (4.38)$$

Inserting this result into the Equation (4.37) and attaining the $dy/dx$ term as the subject of the formula, the change of depth with respect to the horizontal distance in non-uniform flow computations for subcritical flow can be derived as.
If the differential equation of \( y \) with respect to \( x \) (\( dy/dx \)) can be written in terms of change in water depth per change in flow distance of water \((\Delta y/\Delta x)\) then the horizontal distance away from any hydraulic structure where the effects of structure is negligibly small can be written as:

\[
\Delta x = \frac{\Delta y (1 - Fr^2)}{S_0 - \frac{n^2 u^2}{R^{4/3}}} \tag{4.40}
\]

For the direct step method the following computation procedure can be used;

a) Tabulate the hydraulic properties of the channel, such as hydraulic radius and the bed slope of the river.

b) Determine the water depth at the downstream boundary, where there is a hydraulic structure control with a free flow gate. The depth is calculated by the formula

\[
q = c_d a \sqrt{2gy_1}
\]

where \( q \) is the discharge of water, \( c_d \) is the contraction coefficient, \( a \) is the gate area, \( g \) is the gravitational acceleration and \( y_1 \) is the water depth at the structure. \( C_d \) is always smaller than one because there is always an energy loss when there is a flow through a gate.

c) Determine the water depth at the upstream boundary, where the water depth is at the normal levels. This depth is simply calculated by the Mannings Equation.

\[
Q = \frac{A}{n} R^{2/3} S_0^{1/2}
\]

where \( Q \) is the discharge, \( A \) is the cross-sectional area, \( R \) is the hydraulic radius, \( n \) is the manning's roughness coefficient, and \( S_o \) is the river bed slope.

d) Divide total change of depth between end sections into convenient steps. The depth changes are always preferable to be the same, such as, \( \Delta y \).

e) Calculate \( \Delta x \) values between adjacent sections by using Equation (4.40). Later, sum the \( \Delta x \) values to obtain the total length of the channel profile.

Question 4-15

A concrete flood detention wall having 10 circular outlets is constructed on a rectangular channel. The design discharge of the channel is on the average taken as 350 m\(^3\)/sec. This study has executed for various scenarios using different input data. The aim is to understand the
behavior of upstream water depth under the construction of a flood detention structure. The effects of the depth increments, $\Delta y$, the effects of manning’s roughness coefficient $n$, River bed slope, $S_0$, the diameters of the gate openings of the flood detention wall are all to be considered. The contraction coefficient for the gate, is taken as 0.85. For the solution of the problem the Direct Step Method is used. The initial data about the river is given in the table below.

### Table: Initial Data About the River

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge, Q</td>
<td>350 m$^3$/sec</td>
</tr>
<tr>
<td>River Width, b</td>
<td>120 m.</td>
</tr>
<tr>
<td>Gate Width, D</td>
<td>2 m.</td>
</tr>
<tr>
<td>River Slope, $S_0$</td>
<td>0.0008</td>
</tr>
<tr>
<td>Roughness, n</td>
<td>0.02</td>
</tr>
<tr>
<td>Depth Change, $\Delta y$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Solution 4.15

The water depths at the upstream and the downstream boundary of the channel must be calculated before the direct step method calculations.

$$Q = \frac{A}{n} R^{2/3} S_0^{1/2}$$

$$350 = \frac{120y}{0.02} \left( \frac{120y}{120 + 2y} \right)^{2/3} 0.0008^{1/2}$$

$$y = 1.55m.$$  

Therefore, the upstream water depth in the channel is 1.55 meters. On the other hand the downstream depth can be found by

$$Q = c_s a (2gy)^{1/2}$$

$$350 = 0.85 \pi 10 (19.62 y)^{1/2}$$

$$y = 8.75 m$$

which is the water depth at a concrete flood detention wall.

The computational solutions for the Direct Step Method can be solved by the help of Microsoft Excel. The result and the final surface profile is given in the following table and picture.
A concrete flood detention wall having 10 circular outlets is constructed on a rectangular channel. The design discharge of the channel is on the average taken as 350 m$^3$/sec. This study has executed for various scenarios using different input data. The aim is to understand the behavior of upstream water depth under the construction of a flood detention structure. The effects of the depth increments, $\Delta y$, the effects of manning's roughness coefficient $n$, River bed slope, $S_0$, the diameters of the gate openings of the flood detention wall are all to be considered. The contraction coefficient for the gate, is taken as 0.85. For the solution of the problem use the Direct Step Method. The initial data about the river is given in the table below.

<table>
<thead>
<tr>
<th>parameter</th>
<th>magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge, Q</td>
<td>350 m$^3$/sec</td>
</tr>
<tr>
<td>River Width, b</td>
<td>120 m.</td>
</tr>
<tr>
<td>Gate Width, D</td>
<td>2 m.</td>
</tr>
<tr>
<td>River Slope, $S_0$</td>
<td>0.0008</td>
</tr>
<tr>
<td>Roughness, n</td>
<td>0.02</td>
</tr>
<tr>
<td>Depth Change, $\Delta y$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Solve the problem for various scenarios to investigate:
- the effect of amount of depth increments, $\Delta y$, to the upstream flow.
- the effect of the width, $b$, of the channel to the upstream flow.
- the effect of the slope, $S_0$, to the upstream flow.
- the effect of the roughness coefficient, $n$, to the upstream flow.
- the effect of the diameter of outlet to the upstream flow.

The scenarios will be as follows:

<table>
<thead>
<tr>
<th>parameter</th>
<th>magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge, Q</td>
<td>350 m$^3$/sec</td>
</tr>
<tr>
<td>River Width, b</td>
<td>120 m.</td>
</tr>
<tr>
<td>Gate Width, D</td>
<td>2 m.</td>
</tr>
<tr>
<td>River Slope, $S_0$</td>
<td>0.0008</td>
</tr>
<tr>
<td>Roughness, n</td>
<td>0.02</td>
</tr>
<tr>
<td>Depth Change, $\Delta y$</td>
<td>0.1, 0.25, 0.5m</td>
</tr>
<tr>
<td>parameter</td>
<td>magnitude</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------------------------------</td>
</tr>
<tr>
<td>Discharge, Q</td>
<td>350 m$^3$/sec</td>
</tr>
<tr>
<td>River Width, b</td>
<td>100, 120, 140 m.</td>
</tr>
<tr>
<td>Gate Width, D</td>
<td>2 m.</td>
</tr>
<tr>
<td>River Slope, $S_0$</td>
<td>0.0008</td>
</tr>
<tr>
<td>Roughness, n</td>
<td>0.02</td>
</tr>
<tr>
<td>Depth Change, $\Delta y$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>parameter</th>
<th>magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge, Q</td>
<td>350 m$^3$/sec</td>
</tr>
<tr>
<td>River Width, b</td>
<td>120 m.</td>
</tr>
<tr>
<td>Gate Width, D</td>
<td>2 m.</td>
</tr>
<tr>
<td>River Slope, $S_0$</td>
<td>0.0001, 0.0004, 0.0008</td>
</tr>
<tr>
<td>Roughness, n</td>
<td>0.02</td>
</tr>
<tr>
<td>Depth Change, $\Delta y$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>parameter</th>
<th>magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge, Q</td>
<td>350 m$^3$/sec</td>
</tr>
<tr>
<td>River Width, b</td>
<td>120 m.</td>
</tr>
<tr>
<td>Gate Width, D</td>
<td>2 m.</td>
</tr>
<tr>
<td>River Slope, $S_0$</td>
<td>0.0008</td>
</tr>
<tr>
<td>Roughness, n</td>
<td>0.01, 0.02, 0.03</td>
</tr>
<tr>
<td>Depth Change, $\Delta y$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>parameter</th>
<th>magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discharge, Q</td>
<td>350 m$^3$/sec</td>
</tr>
<tr>
<td>River Width, b</td>
<td>120 m.</td>
</tr>
<tr>
<td>Gate Width, D</td>
<td>1.8, 2.0, 2.2 m.</td>
</tr>
<tr>
<td>River Slope, $S_0$</td>
<td>0.0008</td>
</tr>
<tr>
<td>Roughness, n</td>
<td>0.02</td>
</tr>
<tr>
<td>Depth Change, $\Delta y$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Show your results both in tabular form and graphical representation. Also, explain your findings for each different case.
4.11 Design of channels for uniform flow

The channels are divided into:
- non-erodible channels (channels with fixed boundaries.)
- Erodible channels (channels with movable boundaries.)

The non-erodible channels.

Most lined channels and built up channels can withstand erosion satisfactorily and are therefore considered as non-erodible. For non erodible channels the designer simply computes the dimensions of the channel by a uniform flow formula and then decides the final dimensions on the basis of hydraulic efficiency, practicability, and economy. For the channel design the best hydraulic section is the target result of design procedure.

The determination of section dimensions for nonerodible channels includes the following steps;

a) Collect all necessary information, estimate $n$, and select $S_o$

b) Compute the section factor $z = AR^{2/3} = nQ/(S_o)^{1/2}$

c) In the case of the best hydraulic section, substituting the corresponding values of $A$ and $R$ obtained from Table 4.2 into above equation. Unknowns are $b$ and $y$, by assuming several values of $b$, a number of combinations of section dimensions can be obtained. The final dimensions should be determined on the basis of hydraulic efficiencies, and economy. The optimum side slope of best hydraulic section for a trapezoidal channel is $60^\circ$, which is quite steep and creates stability problems.

d) In order to have a stable flow conditions in trapezoidal channels, the critical flow depth ($y_c$) should be determined using Fig 4.10. The condition $y => 1.1y_c$ for subcritical flow or $y <= 0.9y_c$ for supercritical flow should be satisfied. Otherwise, the dimensions of the channel cross-section should be changed to satisfy the criteria.

e) Cross-sectional flow velocity is compared with the minimum allowable velocity against silting in the channel, which is around 0.5 m/sec.

f) Add a proper freeboard to the depth of the channel section.
The freeboard of the channel is the vertical distance from the water surface at the design condition to the top of the channel. This distance should be sufficient to prevent waves or fluctuations in water surface from overflowing the sides. An empirical equation for freeboard determination is given as

\[ f = 0.2(1 + y) \]  \hspace{1cm} (4.41)

where the \( f \) value stands for freeboard in meters and \( y \) is the water depth.

There are two methods of approach to the proper design of erodible channels

i. the method of permissible velocity

ii. the method of tractive force

**Method of Permissible Velocity.**

The maximum permissible velocity or the non-erodible velocity is the greatest mean velocity that will cause no erosion at the channel body. Using the maximum permissible velocities as a criterion, the design procedure for a channel section, assumed to be trapezoidal, consists of the following steps.

a) Determine roughness coefficient, \( n \) (Table 4.4)
b) Side slope, \( z \) (Table 4.5)
c) Maximum permissible velocity \( u_{\text{max}} \)
d) From \( u = (1/n)R^{2/3}S^{1/2} \) (manning's equation) compute the hydraulic radius \( R \).
e) Calculate the flow area, \( A = Q/u_{\text{max}} \)
f) Calculate wetted perimeter, \( P=A/R \)
g) With known \( A \) and \( P \) use table 4.2 and calculate \( b \) and \( y \)

If \( y \neq 1 \) meters, the maximum permissible velocity determined at level 1 must be corrected by using Figure 4.16, and the procedure is repeated again until level \( f \). Critical flow conditions are checked and freeboard is computed.
Table 4.5 Recommended side slopes for unlined channels.

<table>
<thead>
<tr>
<th>Material</th>
<th>Side slope (H:V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>Nearly vertical</td>
</tr>
<tr>
<td>Muck and peat soils</td>
<td>0.25:1</td>
</tr>
<tr>
<td>Stiff clay or earth with concrete lining</td>
<td>0.5:1 to 1:1</td>
</tr>
<tr>
<td>Firm clay or earth for small ditches</td>
<td>1.5:1</td>
</tr>
<tr>
<td>Loose sandy earth</td>
<td>2:1</td>
</tr>
<tr>
<td>Sandy loam or porous clay</td>
<td>3:1</td>
</tr>
</tbody>
</table>

**Permissible Tractive Force Approach.**

When water flows in a channel, a force is developed that acts in the direction of flow on the channel bed. This force, which is simply the pull of water on the wetted area, is known as the tractive force, which is also known as shear force or drag force. The average value of the tractive force per unit wetted area is obtained as:

\[
\tau_0 = \gamma R S_0
\]  

(4.42)

Where, \( \gamma \) is the unit weight of water, \( R \) is the hydraulic radius and \( S_0 \) is the bed slope. In a wide open channel, the hydraulic radius is equal to the depth of flow, \( y \). It should be noted that the unit tractive force is not uniformly distributed along the wetted perimeter. A typical distribution of unit tractive force in a trapezoidal channel is shown in Figure 4.11 and the Figure 4.12 shows the maximum unit tractive forces on the sides and bottom of various channel sections.

Figure 4.11 Distribution of unit tractive force in a trapezoidal channel section with \( b=4y \)

Figure 4.12 Maximum unit tractive forces in terms of \( \gamma y S_0 \)

Permissible tractive force is the maximum unit tractive force that will not cause serious erosion of the material forming the channel bed on a level surface. For non cohesive materials the
The permissible tractive force is a function of the average particle diameter (Figure 4.13) and for cohesive materials it is a function of the void ratio (Figure 4.14). If the channel is designed with non-cohesive material than either bottom or sides are critical. That means the permissible tractive force is a function of the diameter of the particles. However, if the channel is designed with cohesive material than only the bottom is critical. That means the permissible tractive force is a function of the void ratio.

Figure 4.13 Unit tractive forces for canals in non-cohesive material

Figure 4.14 Unit tractive forces for canals in cohesive material
**Design procedure for Permissible Tractive Force Approach.**

For a given design discharge $Q$, type of soil, and a selected channel bed slope $S_0$: Side slope $z$, roughness coefficient, $n$, and angle of repose, $\theta$ are selected. The permissible tractive force, $\tau_p$, is determined either from Figures (4.12) or (4.13) Any $(b/y)$ ratio is assumed and the tractive force of water; on the channel bed; $C_1$ and on the sides of the channel, $C_2$ are determined from Figure (4.12) The value of $y$ is determined from the following inequalities and its smaller value is accepted. In the following equations, $K$ is the tractive force ratio.

\[ C_1 y S_0 \leq \tau_p \]  
\[ (4.43) \]

\[ C_2 y S_0 \leq \tau_p K \]  
\[ (4.44) \]

Using the assumed $(b/y)$ ratio and the computed value of $y$, the channel capacity is determined by the manning equation. If the computed channel capacity is different from the design discharge, a new value for $(b/y)$ ratio is assumed and the procedure is repeated until the computed discharge is equal to the given design discharge. Critical flow conditions are checked and freeboard is added to the water depth.

*Note that for cohesive soils, only the tractive force of bottom is critical. For non cohesive soils, one must compute the value of $K$ from* 

\[ K = \sqrt{1 - \frac{\sin^2 \phi}{\sin^2 \theta}} \]  
\[ (4.45) \]

On the basis of the stability criteria if $K < 0.78$, the side traction is critical and otherwise the bottom traction is critical.

Finally, the section of an erodable channel in which no erosion will occur at a minimum flow area for a given discharge is called the stable hydraulic section.

---

**Question 4-17**

A trapezoidal channel which carries $11.5 \, \text{m}^3/\text{sec}$ discharge is built with concrete lining. Mannings roughness coefficient is 0.025. The channel bed slope is 0.0016 and $z=1.5$. Determine the section dimensions.

**Solution 4-17**

\[ AR^{2/3} = \frac{nQ}{\sqrt{S_0}} = \frac{0.025*11.5}{\sqrt{0.0016}} = 7.19 \]

The expressions for $A$ and $R$ are as

\[ A = by + zy^2 \]
\[ R = \frac{by + zy^2}{b + 2y\sqrt{1 + z^2}} \]
then

\[
\frac{(by + zy^2)^{5/3}}{(b + 2y\sqrt{1 + z^2})^{2/3}} = 7.19 \quad \text{where } z=1.5
\]

In a preliminary approach, the bottom width may be taken as \( b=2.5 \) meters. The assumption is just based on the Figure given as;

![Figure 4.15 Bottom width and depth relationship with the capacity of channel](image)

will give \( y=1.56 \) meters. Let's check the critical flow conditions.

\[
\frac{Q}{\sqrt{gb^{2.5}}} = \frac{11.5}{\sqrt{9.81(2.5)^{2.5}}} = 0.37 \quad \text{from Fig 4.10} \quad \frac{y}{b} = 0.4
\]

\[
y_c = 0.4*2.5 = 1.0 \text{ meters.}
\]

\[
\frac{y}{y_c} = \frac{1.56}{1.0} = 1.56 \geq 1.1 \quad \text{OK!}
\]

mean flow velocity will be calculated from continuity equation by;

\[
u = \frac{Q}{A} = \frac{11.5}{2.5*1.56 + 1.5*1.56^2} = 1.52 \text{ m/sec} > 0.5 \text{ m/sec} \quad \text{OK! no silting}
\]

**Question 4-18**

Compute the bottom width and the depth of flow of an unlined trapezoidal channel laid on a slope of 0.0016 and carrying a discharge of 10m³/sec. The channel is to be excavated in earth consisting of noncolloidal coarse gravel and pebbles. Solve the problem by maximum permissible velocity approach. (n=0.025 and \( u_{\text{max}}=1.35\)m/sec)

**Solution 4-18**

For the given conditions, the following are estimated.
by the manning equation for velocity;

\[ R^{2/3} = \frac{u_{\text{max}} R}{\sqrt{S_0}} = \frac{1.35 \times 0.025}{\sqrt{0.0016}} = 0.843 \]

\[ R = 0.775 \text{ meters.} \]

\[ A = \frac{Q}{u_{\text{max}}} = \frac{10}{1.35} = 7.41 \text{ m}^2 \]

\[ P = \frac{A}{R} = \frac{7.41}{0.775} = 9.56 \text{ m.} \]

\[ P = b + 2y\sqrt{1 + z^2} = b + 4.47y = 9.56 \text{ m.} \]

\[ A = by + zy^2 = A = by + 2y^2 = 7.41 \text{ m}^2 \]

Simultaneous solution of these equations gives \( y = 1.07 \text{ meter and } b = 4.78 \text{ meter}. \) Note that the second root of the quadratic equation \( y = 2.72 \text{ meters which gives a negative value for } b \) and hence it is not valid. Since \( y \) is very close to 1.0 meter, no correction is needed for \( u_{\text{max}}. \)

Figure 4.16 Correction factor for average depth in permissible velocity approach

\[ \frac{Q}{\sqrt{gb^{2.5}}} = \frac{10}{\sqrt{9.81(4.78)^{2.5}}} = 0.064 \quad \text{from fig 4.10} \quad \frac{y}{b} = 0.15 \]
Question 4-19

Compute the bottom width and the depth of flow of an unlined trapezoidal channel laid on a slope of 0.0016 and carrying a discharge of 10 m³/sec. The channel is to be excavated in earth consisting of non-colloidal coarse gravel and pebbles. The roughness coefficient is 0.02. Solve the problem by maximum permissible velocity approach if \( u_{\text{max}} = 1 \) m/sec.

Solution 4-19

\[ R^{2/3} = \frac{u_{\text{max}} n}{\sqrt{S_0}} = \frac{1 \times 0.02}{\sqrt{0.0016}} = 0.5 \quad R = 0.35 \text{ meters}. \]

\[ A = \frac{Q}{u_{\text{max}}} = \frac{1.0}{1.0} = 1.0 \text{ m}^2 \]

\[ P = \frac{A}{R} = \frac{1.0}{0.35} = 2.86 \text{ meters} \]

\[ P = b + 2y\sqrt{1 + z^2} = b + 2.83y = 2.86 \]
\[ A = by + zy^2 = A = by + y^2 = 1 \text{ m}^2 \]

Simultaneous solution of these equations gives \( y = 0.53 \) meter and \( b = 1.36 \) meter. Since \( y = 0.53 < 1.0 \) meters a correction factor to \( u_{\text{max}} \) is needed. Correction factor for \( y = 0.53 \) meters is obtained as C.F. = 0.88 from figure 5.7

\[ u_{\text{max}} = 0.88 \times 1.0 = 0.88 \text{ m/sec} \]

The successive calculations are repeated using new permissible velocities.

<table>
<thead>
<tr>
<th>( u_{\text{max}} )</th>
<th>R (m)</th>
<th>A (m²)</th>
<th>P (m)</th>
<th>y (m)</th>
<th>b (m)</th>
<th>C.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.88</td>
<td>0.29</td>
<td>1.14</td>
<td>3.93</td>
<td>0.35</td>
<td>2.94</td>
<td>0.82</td>
</tr>
<tr>
<td>0.82</td>
<td>0.26</td>
<td>1.22</td>
<td>4.69</td>
<td>0.3</td>
<td>3.84</td>
<td>0.80</td>
</tr>
<tr>
<td>0.80</td>
<td>0.25</td>
<td>1.25</td>
<td>5.00</td>
<td>0.28</td>
<td>4.21</td>
<td>0.80</td>
</tr>
</tbody>
</table>

so section dimensions become \( y = 0.28 \) and \( b = 4.21 \) meters.
\[
\frac{Q}{\sqrt{gb^{2.5}}} = \frac{1.0}{\sqrt{9.81(4.21)^{2.5}}} = 0.0088 \quad \text{from fig 4.10} \quad \frac{y}{b} = 0.042
\]

\[y_c = 0.042 \times 4.21 = 0.18 \text{ m.}
\]

\[\frac{y}{y_c} = \frac{0.28}{0.18} = 1.56 \geq 1.1 \text{ OK!}
\]

Earth freeboard \( f = 0.2(1 + y) = 0.2(1.28) = 0.26 \) meters