CE582
CREEP & SHRINKAGE OF CONCRETE

LECTURE NOTES
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ELASTICITY & CREEP

ELASTICITY

**Pure Elasticity**: strains appear and disappear immediately on application and removal of stress.

- **Fig.1** (see next slide) different elastic-plastic behaviours.
Fig. 1

(a) is linear and elastic,
(b) is non-linear and elastic.

*Steel conforms to case (a).*

*Some plastics and timber follow case (b).*

Brittle materials such as glass and most rocks conforms to case (c).

The fourth category (d) can be described as non-linear and non-elastic behavior, a permanent deformation existing after removal of load.

This behavior is typical of *concrete in compression or tension* loaded to moderate and high stresses but is not very pronounced at very low stresses.

![Fig. 1 Categories of stress-strain response](image)
Modulus of elasticity

- The slope of stress-strain curve gives the modulus of elasticity.

- Young’s modulus can be determined for the initial part of the above curve Fig. 1(d).

- If there is no straight portion of the curve, the tangent to the curve at the origin can be measured. This is **INITIAL TANGENT MODULUS**.

- It is also possible to find a **TANGENT MODULUS** at any point on the stress-strain curve.

Typical stress–strain curve for concrete

Note: In dry concrete a small concave part of the curve at the beginning of loading in compression is sometimes encountered due to the existence of fine shrinkage cracks.
The magnitude of the observed strains and the curvature of the stress-strain relation depend to some extent on the rate of application of stress.

If the load applied is extremely rapid (< 0.01 sec), recorded strains are greatly reduced and the curvature of the stress-strain curve becomes very small.

About 10 minutes is required to test a specimen in an ordinary testing machine-the increase in strain and hence the degree of non-linear behavior are very small.
SECANT MODULUS

• **SECANT MODULUS** is measured at stresses ranging from 15% to 50% of the short term strength (there is no standard method to determine).

• Initial Tangent Modulus is approximately equal to the Dynamic Modulus.

• **ASTM C469** describes determination of static modulus of elasticity.

• **Chord Modulus (rarely used)** = Static modulus
Table 1. Typical range of values of 28 days static modulus of elasticity for normal weight concrete, according to BS 8110:Part 2.

<table>
<thead>
<tr>
<th>28 – Day cube compressive strength (MPa)</th>
<th>Mean 28-Day static modulus of elasticity (GPa)</th>
<th>Typical range of 28-day static modulus of elasticity (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>24</td>
<td>18 to 30</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>19 to 31</td>
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<tr>
<td>30</td>
<td>26</td>
<td>20 to 32</td>
</tr>
<tr>
<td>40</td>
<td>28</td>
<td>22 to 34</td>
</tr>
<tr>
<td>50</td>
<td>30</td>
<td>24 to 36</td>
</tr>
<tr>
<td>60</td>
<td>32</td>
<td>26 to 38</td>
</tr>
</tbody>
</table>
The static modulus of elasticity (Ec) in GPa can be related to cube compressive strength $f_{cu}$ (MPa) by the expression:

$$E_c = 9.1 \cdot f_{cu}^{0.33}$$  \hspace{1cm} (1)

When the density of concrete, $\rho = 2320$ kg/ m$^3$, i.e. typical normal weight concrete.

When $\rho = 1400$-2320 kg/ m$^3$

$$E_c = 1.7 \cdot f_{cu}^{0.33} \cdot 10^{-6}$$  \hspace{1cm} (2)
The ACI Building code 318-89 gives the following expression for normal weight concrete;

\[ E_c = 4.7 \ f_{cyl}^{0.5} \]  \hspace{1cm} (3)

\( f_{cyl} \): Cylinder compressive strength (MPa)

When density of concrete is between 1500-2500 kg/ m\(^3\), the static modulus of elasticity is given as;

\[ E_c = 43 \ \rho^{1.5} \ f_{cyl}^{0.5} \ 10^{-6} \]  \hspace{1cm} (4)
Dynamic Modulus of Elasticity

BS1881:Part 209: 1990 and ASTM C215
Specimens similar to flexural strength specimens.

Beams: 150x150x750 mm or 100x100x400 mm

\[ E_d = 4 \, n^2 \, L^2 \, \rho \times 10^{-6} \]  \hspace{1cm} (5)

n: Frequency (hz)
L: length of specimen (mm)
\( \rho \): Density of concrete (kg/ m\(^3\))
FACTORS INFLUENCING MODULUS OF ELASTICITY

1. Moisture Condition

2. Properties of Aggregates

Greater the volume of aggregates the higher the modulus of concrete.

Relation between modulus of elasticity of concrete and strength depends on the age. Modulus increases more rapidly than strength.

*Modulus of elasticity (LWA) = 40-80% Modulus of elasticity (NWA)*

*Strength should be same.*

Static modulus of elasticity is affected by the shape of stress-strain curve for concrete as determined in laboratory.
POISSON’S RATIO ($\mu$):

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$\mu = 0.15 – 0.20$ (Normal weight concrete and lightweight concrete)

Dynamic Modulus of Elasticity:

- Velocity of pulse of ultrasonic waves and the fundamental resonant frequency of longitudinal vibration of a concrete beam specimen is measured.

ASTM C215.

- **Fig. 3** Test arrangement for the determination of the dynamic modulus of elasticity (longitudinal vibration).

- Pulse is obtained by ASTM C597.
Fig. 3 Test arrangement for the determination of the dynamic modulus of elasticity (longitudinal vibration) given by BS 1881: Part 209.
Then, $\mu$ can be obtained from the following relation.

$$\left( \frac{V}{2NL} \right)^2 = \frac{1 - \mu}{(1 + \mu)(1 - 2\mu)}$$

(8)

$V$: Pulse velocity (mm/sec)
$N$: Resonant frequency (hz)
$L$: length of the beam (mm)
$\mu$ obtained is in the range of 0.2-0.24
The relation between stress and strain is a function of time.

**Creep** is defined as the increase in strain under a sustained constant stress after taking into account other time-dependent deformation not associated with stress, viz. shrinkage, swelling and thermal deformation.

Consider the following: Concrete loaded to a compressive stress $\sigma_0$ at the age $t_0$ and subjected to the same stress $\sigma_0$ until some time $t$ ($t>t_0$).

It is assumed that the concrete has been cured in water until age $t_0$ and subsequently stored in various environments.

The secant modulus of elasticity at the age $t_0$ was determined and is referred to as E.

Hence, the elastic strain is $\sigma_0/E$.

*Fig. 4* Definition of creep under a constant stress $\sigma_0$; E is the secant modulus of elasticity at age $t_0$
Definition of creep under a constant stress $\sigma_0$; $E$ is the secant modulus of elasticity at age $t_0$. 
Conditions of creep behavior:

(a) Concrete sealed from the environment from the age \( t_0 \).

At the age \( t \), the measured strain \( \varepsilon_a \) is comprised of initial elastic strain \( \sigma_0/E \) and creep \( C_a \). Hence,

\[
C_a = \varepsilon_a - \frac{\sigma_0}{E} \quad \text{(9)}
\]

(b) Concrete allowed to dry from the age \( t_0 \).

At age \( t \), the measured strain \( \varepsilon_b \) is comprised of the same initial elastic strain as before and of creep \( C_b \) and shrinkage \( S_h \). Since shrinkage is a contraction, we have;

\[
C_b = \varepsilon_b - \frac{\sigma_0}{E} - S_h \quad \text{(10)}
\]
(c) **Concrete stored in water from the age \( t_0 \).**

At age \( t \), the measured strain (\( \varepsilon_c \)) is comprised of the same initial elastic strain as before and creep (\( C_c \)) and swelling (\( S_w \)). Swelling is, by definition, an expansion, so that,

\[
C_c = \varepsilon_c - \frac{\sigma_0}{E} - S_w
\]  

(11)

(d) **Concrete sealed from the age \( t_0 \) and subjected to a rise in temperature.**

At the age \( t \), the measured strain is comprised of the same elastic strain as before of creep (\( C_d \)) and of thermal expression (\( S_T \)). Hence,

\[
C_d = \varepsilon_d - \frac{\sigma_0}{E} + S_T
\]  

(12)
If a loaded concrete specimen is restrained so that it is subjected to a constant strain; creep will manifest itself as a progressive decrease in stress with time.

This is termed as Relaxation.

Fig. 5 Definition of relaxation for concrete subjected initially to stress $\sigma_0$ and kept at a constant strain; $E$ is the secant modulus of elasticity at age $t_0$. 
If a sustained load is removed after some time, the strain decreases immediately by an amount equal to the elastic strain. This strain is smaller than the initial elastic strain because of the increase in the modulus of elasticity with age.

The **INSTANTANEOUS** recovery is followed by a gradual decrease in strain, called **CREEP RECOVERY**.

The creep recovery is always smaller than the preceding creep so that there is a **RESIDUAL DEFORMATION** (even after a period under load of one day only).

Knowledge of creep recovery is of interest in connection with estimating stresses when relaxation occurs (i.e. prestressed concrete).

**Fig. 6** Creep and creep recovery of concrete stored in water and in air from the age of 28 days, subject to a stress of 9 MPa and then unloaded; mix proportions 1: 1.7: 3.5 by mass; water/cement ratio of 0.5; specimen size 75x255 mm cylinder; cured in water.
FACTORS INFLUENCING CREEP

1. Modulus of elasticity of aggregate.

*Fig. 7* Effect of modulus of elasticity of aggregate or relative creep of concrete (equal to 1 for an aggregate with a modulus of elasticity of 69 GPa)
Fig. 7 Effect of modulus of elasticity of aggregate or relative creep of concrete (equal to 1 for an aggregate with a modulus of elasticity of 69 GPa)
FACTORS INFLUENCING CREEP

2. Volume of aggregate:
   • Higher volume gives lower creep.
   • $g =$ aggregate content by vol.
   • Cement paste content $= 100 - g$

Fig. 8 Effect of volumetric content of aggregate on creep of concrete, corrected for variations in the w/c ratio.
Fig. 8 Effect of volumetric content of aggregate on creep of concrete, corrected for variations in the w/c ratio.
FACTORS INFLUENCING CREEP

3. Water to cement ratio:

- Lower w/c ratio gives higher strength.

- Fig. 9 Data of several investigators adjusted for the volumetric content of cement paste (to a value of 0.20), with creep expressed relative to the creep at a w/c ratio of 0.65.

4. Age of application of Load

- Creep decreases as the age of application of load increases, because strength increases.

- Fig.10 Influence of age at application of load on creep of concrete relative to creep of concrete loaded at 7 days, for tests of different investigators; concrete stored at a relative humidity of approximately 75%.
**Fig. 9** Data of several investigators adjusted for the volumetric content of cement paste (to a value of 0.20), with creep expressed relative to the creep at a w/c ratio of 0.65.
Fig. 10 Influence of age at application of load on creep of concrete relative to creep of concrete loaded at 7 days, for tests of different investigators; concrete stored at a relative humidity of approximately 75%.
FACTORS INFLUENCING CREEP

5. Relative Humidity

• Relative humidity of the air surrounding concrete will affect the creep of concrete.

• Creep is higher, the lower the relative humidity.

• Fig. 11 Creep of concrete cured in fog for 28 days and then loaded and stored at different relative humidities.

• The influence of R.H. on creep and on shrinkage is similar and both deformation are also dependent on the size of the concrete members.
**Fig. 11** Creep of concrete cured in fog for 28 days and then loaded and stored at different relative humidities.
FACTORS INFLUENCING CREEP

6. Size of Member

• When drying occurs at a constant R.H., Creep is smaller in a larger specimen, this size effect is expressed later on.
• If no drying occurs, as in the mass concrete, creep is smaller and is independent of size because there is no additional effect of drying on creep.

7. Temperature

• Concrete in nuclear pressure vessels
• Bridges
• The time at which the temperature of concrete rises relative to the time at which the load is applied affects the creep-temperature relation.
• If saturated concrete (simulated mass concrete) is heated and loaded at the same time, creep is greater than when concrete is heated during the curing period, prior to application of load (see fig below)
• Fig. 12 Influence of temperature of creep of saturated concrete relative to creep at 21 C; specimens cured at the stated temperature from 1 day until loading at 1 year.
Fig. 12 Influence of temperature of creep of saturated concrete relative to creep at 21°C; specimens cured at the stated temperature from 1 day until loading at 1 year.
FACTORS INFLUENCING CREEP

• Creep is smaller when concrete is cured at a high temperature because strength is higher than when concrete is cured at normal temperature before heating and loading.

• Figure 13 below shows creep at low temperature as a proportion of creep at 20 C, creep decreases until the formation of ice which causes an increase in creep but below the ice point, creep again decreases.
**Fig. 13** Creep of sealed concrete at low temperature relative to creep at 20°C
FACTORS INFLUENCING CREEP

8. Cement Type

• Cement type influences the strength of concrete at the time of application of load.
• Most PC lead to sensibly the same creep.
• Specific creep increases in the order of type of cement: HAC, RHPC, OPC.
• The order of magnitude of creep of Portland BFSC, Low heat PC and Portland Pozzolan cement is less clear.

**If there is drying, creep becomes higher.
Magnitude of Creep

- The presence of several factors influencing creep makes it impossible to quote reliable typical values.

- For practical purposes, we are usually interested in creep after several months or years, or even in the ultimate (or limiting) value of creep. We know that the increase in creep beyond 20 years under load (within the range of working stresses) is small, and as a guide, we can assume that:

  - 25% of the 20 year creep occurs in 2 weeks
  - 50% of the 20 year creep occurs in 3 weeks
  - 75% of the 20 year creep occurs in 1 year

- Several methods of estimating creep are available but with unknown materials, it may be necessary to determine creep of concrete by experiment. ASTM C512 describes a test method.
Magnitude of Creep

Typical equations relating creep after any time under load, \( C_t \), to creep after 28 days under load, \( C_{28} \) are:

For sealed or saturated concrete:

\[
C_t = C_{28} 0.5 t^{0.2}
\]  \( (9) \)

For drying concrete:

\[
C_t = C_{28}[-6.19 + 2.15 \log_e t]^{1/2.65}
\]

\( t \): time under load (days) > 28 days

The above expressions are sensibly independent of mix proportions, type of aggregates, size of specimens and age at loading.
PREDICTION OF CREEP

a) Normal weight concrete, constant stress, stored under normal constant environmental conditions.

ACI 209-R-82, creep coefficient \( \Phi(t,t_o) \), as a function of time:

\[
\phi(t,t_o) = \frac{(t-t_o)^{0.6}}{10 + (t-t_o)^{0.6}} \phi_\infty(t_o) \tag{10}
\]

\[
\phi(t,t_o) = c(t,t_o)E_c(t_o) \tag{11}
\]

\[
\phi(t,t_o) = 2.35k_1k_2k_3k_4k_5k_6 \tag{12}
\]

\( \phi_\infty(t,t_o) \): Ultimate creep coefficient

\( E_c \): Modulus of elasticity.
For ages at applied load greater than 7 days for moist curing, or greater than 1 to 3 days for steam curing, the coefficient $k_1$ is estimated from:

For moist curing: $k_1 = 1.25 t_o^{-0.118}$  \hspace{1cm} (13)
For steam curing: $k_1 = 1.13 t_o^{-0.095}$  \hspace{1cm} (14)

The coefficient $k_2$ is dependent upon the relative humidity (h)-%:
$k_2 = 1.27 - 0.006 h$ (for $h \geq 40$)  \hspace{1cm} (15)
k3 allows for member size in terms of the vol./surface ratio. V/S which is defined as the ratio of the cross-sectional area to the perimeter exposed to drying.

use the following table to find k3;

- **Table 2. Values of k3 for V/S < 37.5 mm**

<table>
<thead>
<tr>
<th>Volume/surface ratio</th>
<th>Coefficient $k_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>1.3</td>
</tr>
<tr>
<td>19</td>
<td>1.17</td>
</tr>
<tr>
<td>25</td>
<td>1.11</td>
</tr>
<tr>
<td>31</td>
<td>1.04</td>
</tr>
<tr>
<td>37.5</td>
<td>1.00</td>
</tr>
</tbody>
</table>
V/ S: (37.5-95 mm), k₃ is given by:

For (t-t₀) ≤ 1 year:

\[ k₃ = 1.14 - 0.00364 \frac{V}{S} \]  \hspace{2cm} (16)

For (t-t₀) > 1 year:

\[ k₃ = 1.10 - 0.00268 \frac{V}{S} \]  \hspace{2cm} (17)

When V/ S ≥ 95 mm:

\[ k₃ = \frac{2}{3} \left[ 1 + 1.13e^{-0.0212(V/S)} \right] \]  \hspace{2cm} (18)
The coefficients to allow for the composition of the concrete are $k_4$, $k_5$ and $k_6$.

$k_4 = 0.82 + 0.00264(s) \quad \text{in SI units}$  \hspace{1cm} (19)

$s$: slump of fresh concrete in mm.

$k_5$ depends on the fine aggregate/total aggregate ratio, $A_f/A$, in per cent and is given by:

$$k_5 = 0.88 + 0.0024 \frac{A_f}{A}$$  \hspace{1cm} (20)

$k_6$ depends on the air content $a$ (%):

$$k_6 = 0.46 + 0.09(a) \geq 1.$$  \hspace{1cm} (21)
CREEP FUNCTION

The elastic strain-plus-creep deformation under a unit stress is termed the creep function ($\Phi$) and is given by:

$$\Phi(t, t_o) = \frac{1}{E_c(t_o)} \left[ 1 + \phi(t, t_o) \right]$$  \hspace{1cm} (22)

$E_c(t_o)$ is related to the compressive strength of test cylinders by

$$E_c = 43 \rho^{1.5} f_{cyl}^{0.5} 10^{-6}$$ \hspace{1cm} (23)

(In SI units)

If the strength at age $t_o$ is not known, it can be found from the following relation:

$$f_{cyl}(t_o) = \frac{t_o}{X + (Yt_o)} f_{cyl-28}$$ \hspace{1cm} (24)

$f_{cyl-28}$: strength at 28 days

$X, Y$: Given in table below.
Table 3. Values of the constants X and Y using ACI method of prediction of creep

<table>
<thead>
<tr>
<th>Type of cement</th>
<th>Curing condition</th>
<th>Constants of equation 24</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>OPC (Type I)</td>
<td>Moist</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Steam</td>
<td>1</td>
</tr>
<tr>
<td>Rapid hardening PC (Type III)</td>
<td>Moist</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>Steam</td>
<td>0.70</td>
</tr>
</tbody>
</table>
For concrete with an average, high quality dense aggregate the modulus of elasticity $E_c(t_o)$ is related to the compressive strength of cubes, $f_{cu}(t_o)$ as follows:

$$E_c(t_o) = E_{c28} \left[ 0.4 + 0.6 \frac{f_{cu}(t_o)}{f_{cu28}} \right]$$

(25)

The modulus of elasticity at 28 days, $E_{c28}$ is obtained from the cube strength at 28 days $f_{cu28}$ by the following expression:

In GPa: $E_{c28} = 20 + 0.2f_{cu28}$

(26)

For LWA concrete of density $\rho$;

$$E_c(t_o) = \left( \frac{\rho}{2400} \right)^2 E_{c28} \left[ 0.4 + 0.6 \frac{f_{cu}(t_o)}{f_{cu28}} \right]$$

$$E_{c28} = \left( \frac{\rho}{2400} \right)^2 [20 + 0.2f_{cu28}]$$
The strength ratio term of Eqn.25 is best obtained by measurement; however the values of *Table 4* may be used.

<table>
<thead>
<tr>
<th>Age ((t_o))</th>
<th>Strength ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.70</td>
</tr>
<tr>
<td>28</td>
<td>1.00</td>
</tr>
<tr>
<td>90</td>
<td>1.17</td>
</tr>
<tr>
<td>365</td>
<td>1.25</td>
</tr>
</tbody>
</table>

**CE582 Creep & Shrinkage of Concrete**
For very long time under load, the ultimate creep function ($\Phi_\infty$) is given by:

$$\Phi_\infty = \frac{1}{E_c(t_o)}(1 + \phi_\infty)$$  \hspace{1cm} (27)

$\phi_\infty$: ultimate creep coefficient by Fig. given below.
• Given: Ambient RH, age at application of load, V/S ratio.
• Obtained: ultimate creep function.

• If there is no moisture exchange, i.e. the concrete is sealed or we are dealing with mass concrete, creep is assumed to be equivalent to that of concrete with a V/S greater than at 100% RH.

• **Fig. 14** data for estimating the ultimate creep coefficients for use in Eqn.27.
Fig. 14: data for estimating the ultimate creep coefficients for use in Eqn. 27.
DEFORMATION & CRACKING INDEPENDENT OF LOAD

• In addition to deformation caused by the applied stress, volume changes due to shrinkage and temperature variation are of considerable importance and therefore they induce stress.

• The main danger is the presence of tensile stress induced by same form of restraint to these movements. Because concrete is very weak in tension and prone to cracking.

• Cracks must be avoided and controlled and minimized because they impair the durability and structural integrity and also are aesthetically undesirable.
SHRINKAGE & SWELLING

• **Shrinkage**: caused by loss of water by evaporation or by hydration of cement, and also by carbonation.

• The reduction in volume i.e. volumetric strain is equal to 3 times the linear contraction and shrinkage is simply measured as linear strain.
  – Units: mm/mm $\times 10^{-6}$

• **Plastic Shrinkage**: When the cement paste is plastic, it undergoes a volumetric contraction whose magnitude is of the order of 1% of the absolute volume of dry cement. It is caused by:
  – loss of water by evaporation from the surface of concrete.
  – Suction of water by dry concrete below.
• The contraction induces tensile stresses in the surface layers because they are restrained by the non-shrinking inner concrete and since the concrete is very weak in its plastic state, plastic cracking at the surface can readily occur.

Plastic shrinkage depends on,
• -rate of evaporation of $\text{H}_2\text{O}$ and this depends on:
  – a) air temperature
  – b) concrete temperature
  – c) Relative Humidity of the air
  – d) Wind speed

ACI 305.R.91 specifies Evaporation rate $>0.5 \text{ kg/hr/m}^2$ should be avoided to prevent plastic cracking.

Using larger cement content increases plastic shrinkage of concrete, or lowers plastic shrinkage by increasing aggregate content. (by volume)
**Fig. 15** Influence of cement content of the mix on plastic shrinkage in air at 20°C.

*Autogenous Shrinkage*: It occurs even when there is no moisture movement to or from the set concrete.

- This is caused by loss of H₂O used up in hydration and except in mass concrete structures; it is not distinguished from shrinkage of hardened concrete due to loss of water to the outside.
- It is very small, typically 50×10⁻⁶ to 100×10⁻⁶

*Swelling*:

- If there is continuous supply of H₂O to the concrete during hydration, concrete expands due to absorption of water by the cement gel; this is known as swelling.

**DRYING SHRINKAGE**

- Withdrawal of water from hardened concrete stored in unsaturated air causes drying shrinkage.
**Fig. 15** Influence of cement content of the mix on plastic shrinkage in air at 20°C.
• **Fig 16** Moisture movement in concrete (a) concrete which has dried from age to until age t and was re-saturated (b) concrete which has dried from age to until age t and was then subjected to cycles of drying and wetting.

• Fig. 16 shows that if concrete which has been allowed to dry in air of a given R.H, is subsequently placed in H₂O (or a higher humidity) it will swell due to absorption of H₂O by the cement paste. Not all the initial drying shrinkage is, however, recovered even after prolonged storages in H₂O.

• For the usual range of concretes, the reversible moisture movement (or wetting expansion) represents about 40 to 70 per cent of the drying shrinkage, but this depends on the age before the onset of first drying.
Fig. 16 shows that if concrete which has been allowed to dry in air of a given R.H, is subsequently placed in H₂O (or a higher humidity) it will swell due to absorption of H₂O by the cement paste.

Not all the initial drying shrinkage is, however, recovered even after prolonged storages in H₂O.

**Fig. 16** Moisture movement in concrete (a) concrete which has dried from age to until age t and was re-saturated (b) concrete which has dried from age to until age t and was then subjected to cycles of drying and wetting.
• If concrete is cured so that it is fully hydrated before being exposed to drying, then the reversible moisture movement will form a greater proportion of the drying shrinkage.

• **Fig. 16(b)** shows the pattern of moisture movement under alternating wetting and drying a common occurrence in practice.

• The magnitude of this cyclic moisture movement clearly depends upon the duration of the wetting and drying periods. But drying is very much slower than wetting.

• Thus, the consequence of prolonged dry weather can be reversed by a short period of rain.

The movement depends on:
  – Composition of concrete
  – Degree of hydration
• **IRREVERSIBLE** part of shrinkage is associated with the formation of additional physical and chemical bonds in the cement gel when adsorbed water has been removed.

The general pattern of behavior is as follows:

• When concrete dries, first of all, there is the loss of free water (*water in the capillaries which is not physically bound*).

• This process induces internal RH gradients w/in the cement paste structure so that, with time, water molecules are transferred from the large surface area of the calcium silicates hydrates into the empty capillaries and then out of the concrete.

• In consequence, the cement paste contracts but the reduction in volume is not equal to the volume of water does not cause a significant volumetric contraction of the paste and because of internal restraint to consolidation by the calcium silicate hydrate structure.
Mechanism of drying and carbonation shrinkage is different.

Carbonation = reaction of \( \text{CO}_2 \) with hydrated cement.

\( \text{CO}_2 \): present in atmosphere
\( \text{CO}_2 \): 0.03% (rural air)
\( \text{CO}_2 \): 0.1% or > (large cities) (unventilated laboratories)

\( \text{CO}_2 + \text{moisture} \rightarrow \text{Carbonic Acid.} \)

Carbonic Acid + Ca(OH)\(_2\) \( \rightarrow \) CaCO\(_3\)

(other cement compounds are also decomposed with these reactions)

It causes contraction of concrete and known as CARBONATION SHRINKAGE.
• **Carbonation** starts from the surface of concrete but it is very slow.

• Rate depends on:
  • (1) permeability of concrete,
  • (2) its moisture content, and CO₂ content,
  • (3) relative humidity of the ambient medium.

• Concrete with high w/c ratio together with inadequate curing will be more prone to carbonation.

• Higher w/c ratio = higher permeability

• This will lead to a **greater depth** of carbonation
Examination of Carbonation (Test)

- Extend of carbonation can be determined by treating a freshly broken surface with phenolphthalein - the free Ca(OH)$_2$ is colored pink while the carbonated portion is uncolored.

- **Fig. 17** Drying shrinkage and carbonation shrinkage of mortar at different R.H.

- Carbonation neutralizes the alkaline nature of the hydrated cement paste and thus the protection of steel from corrosion is vitiated.

- Consequently, if the full depth of cover of reinforcement is carbonated and moisture and oxygen can ingress, corrosion of steel and possibly cracking will result.
Carbonation at the exposed surface

Non-carbonated paste

Carbonation along a crack

\[
\text{Ca(OH)}_2 + \text{CO}_2 \rightarrow \text{CaCO}_3 + \text{H}_2\text{O}
\]
**Fig. 17** Drying shrinkage and carbonation shrinkage of mortar at different R.H.
FACTORS INFLUENCING SHRINKAGE

(1) Aggregate

For a constant w/c ratio, at a given degree of hydration, the relation between shrinkage of concrete $S_{hc}$ shrinkage of neat cement paste $S_{hp}$, and the relative volume of concentration of aggregate “g” is

$$S_{hc} = S_{hp} (1-g)^n$$

(28)

$n$ = Depends on modulus of elasticity and Poison’s Ration of aggregate and the concrete.
FACTORS INFLUENCING SHRINKAGE

• **Fig. 18** Influence of volume content of aggregate in concrete (by vol.) on the ratio of the shrinkage of concrete to the shrinkage of neat cement paste (n=1.7)

• Increasing aggregate content from 71 to 74% will reduce the shrinkage by about 20% (see fig.)
Fig. 18 Influence of volume content of aggregate in concrete (by vol.) on the ratio of the shrinkage of concrete to the shrinkage of neat cement paste ($n=1.7$)
FACTORS INFLUENCING SHRINKAGE

Type of aggregate:
• Influences shrinkage of concrete so that lightweight concrete exhibits a higher shrinkage than concrete made with normal wt. aggregate.

• [Fig. 19] Shrinkage of concretes of fixed mix proportion but made with different aggregates, and stored in air at 21 C and a relative humidity of 50%.

Cement Paste:
• Its quality influences the magnitude of shrinkage; the higher the w/c ratio the larger the shrinkage. For a given aggregate content, shrinkage of concrete is a function of the w/c ratio (see Fig.)

• [Fig. 20] Influence of w/c ratio and aggregate content on shrinkage.
Fig. 19 Shrinkage of concretes of fixed mix proportion but made with different aggregates, and stored in air at 21 C and a relative humidity of 50%.
Fig. 20 Influence of w/c ratio and aggregate content on shrinkage
FACTORS INFLUENCING SHRINKAGE

**Time**
- In any case, the rate of shrinkage decreases rapidly with time so that, generally
- 14 to 34% of 20-years shrinkage occurs in 2 weeks
- 40 to 80% of 20 years shrinkage occurs in 3 months
- 66 to 85% of 20 years shrinkage occurs in 1 year.

**Relative Humidity**
- R.H. of the air surrounding the concrete the concrete greatly affects the magnitude of shrinkage, as shown in Fig.

- **Fig. 21** Relation between shrinkage and time for concretes stored at different RH.

- BS 1881: Part 5: 1970... Shrinkage test...: The specimens are dried for a specified period under prescribed conditions of temperature and humidity. The shrinkage occurring under these conditions is of the same order as that after a long exposure to air with a RH of approx. 65%, the latter being representative of the average of indoor (45%) and out door (85%) conditions in the UK.
**Fig. 21** Relation between shrinkage and time for concretes stored at different RH.
ACI 209. R-82, shrinkage $S_h(t, \tau_0)$ at time $t$ (days) measured from the start of drying at $(days)$ is expressed as follows.

For moist curing:

$$S_h(t, \tau_0) = \frac{t - \tau_0}{35 + (t - \tau_0)} S_{h\infty}$$  \hspace{1cm} (29)$$

For steam curing:

$$S_h(t, \tau_0) = \frac{t - \tau_0}{55 + (t - \tau_0)} S_{h\infty}$$  \hspace{1cm} (30)$$

Where $S_{h\infty} = \text{ultimate shrinkage, and}$

$$S_{h\infty}(t, \tau_0) = 780 \times 10^{-6} k_1'k_2'k_3'k_4'k_5'k_6'k_7'$$  \hspace{1cm} (31)$$

$k_i'$: curing times different from 7 days (moist cured concrete) use the following table. For steam curing with a period of 1 to 3 days, $k_i' = 1.$
• **PERDICTIION OF DRYING SHRINKAGE & SWELLING**

• **Table 5 Shrinkage coefficient k1`.**

$k_2'$: humidity coefficient.

- $k_2' = 1.40 - 0.010 \ h \ (40 \leq h \leq 80)$  \hspace{1cm} (32)
- $k_2' = 3.00 - 0.03 \ h \ (80 \leq h \leq 100)$

$h$: Relative humidity (%).

$k_2' = 0$ if $h = 100\%$, So ACI method does not predict swelling.

$k_3'$: allows the size of the member in terms of the vol/ surface ratio.

$v/ s$: For values $v/ s < 37.5 \ mm$, $k_3' =$ as given in table below. When $v/ s$ is between 37.5 and 95 mm:

for $(t - \tau_0) \leq 1 \ year$

$$k_3' = 1.23 - 0.006 \frac{v}{S} \hspace{1cm} \text{(in SI units)}$$  \hspace{1cm} (33)
Table 5. Shrinkage coefficient $k_1$

<table>
<thead>
<tr>
<th>Period of moist curing (days)</th>
<th>Shrinkage coefficient ($k_1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>1.1</td>
</tr>
<tr>
<td>7</td>
<td>1.0</td>
</tr>
<tr>
<td>14</td>
<td>0.93</td>
</tr>
<tr>
<td>28</td>
<td>0.86</td>
</tr>
<tr>
<td>90</td>
<td>0.75</td>
</tr>
</tbody>
</table>

PERDICTION OF DRYING SHRINKAGE & SWELLING

Table 6. Shrinkage Coefficient $k_3'$.

for $(t - \tau_0) > 1$ year:

$$k_3' = 1.17 - 0.06 \frac{V}{S}$$  \hspace{1cm} (34)

When $V/ S \geq 95$ mm:

$$k_3' = 1.2e^{-0.00473(V / S)}$$  \hspace{1cm} (35)

$k_4'$: Coef. of concrete composition.

$$k_4' = 0.89 + 0.00264S$$  \hspace{1cm} (36)

$S$ = Slump of fresh concrete in mm.
Table 6. Shrinkage coefficient $k_3`$

<table>
<thead>
<tr>
<th>V/S ratio (mm)</th>
<th>Coefficient $k_3`$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>1.35</td>
</tr>
<tr>
<td>19</td>
<td>1.25</td>
</tr>
<tr>
<td>25</td>
<td>1.17</td>
</tr>
<tr>
<td>31</td>
<td>1.08</td>
</tr>
<tr>
<td>37.5</td>
<td>1.00</td>
</tr>
</tbody>
</table>

total 95
PERDICTION OF DRYING SHRINKAGE & SWELLING

\[ k'_5 = 0.3 + 0.014 \frac{A_f}{A}, \quad (A_f/A \leq 50) \]  
(37)

\[ k'_5 = 0.9 + 0.002 \frac{A_f}{A}, \quad (A_f/A > 50) \]

\[ A/f/A = \text{(Fine aggregate/total aggregate) ratio by mass (\%).} \]

\[ k'_6 = 0.75 + 0.00061 \gamma \]  
(38)

\( \gamma \): Cement content (kg/m³)

\[ k'_7 = 0.95 + 0.008 A \]  
(39)

\( A \): Air content (%)
An improvement in the accuracy of prediction of shrinkage is obtained by undertaking short-term tests of 28 day duration and then extrapolating to obtain long-term values. The following expression is applicable for both normal weight and lightweight concretes, stored in any drying environment at normal temperature.

$$S_h(t, \tau_0) = S_{h28} + 100 \left[ 3.61 \log_e (t - \tau_0) - 12.05 \right]^{1/2}$$

(40)

$S(t, \tau_0)$ = long term shrinkage ($10^{-6}$) at age $t$ after drying from an earlier age

$S_{h28}$ = Shrinkage ($10^{-6}$) after 28 days and

$(t, \tau_0)$ = time since start of drying ($>28$ days)
PERDICTION OF DRYING SHRINKAGE & SWELLING

The use of eqn. (13) leads to an average error of about ±17% when ten-year predicted shrinkage is compared with measured shrinkage.

For prediction of swelling, test duration of at least one yr. is required to estimate long-term swelling with a reasonable accuracy (e.g. an av. Error of ±18% at ten years). The expression is as follows:

\[
S_w(t, \tau_0) = S_{w365}^B
\]

\[
B = 0.377 \left[ \log_e (t - \tau_0) \right]^{0.55}
\]

\[
S_w(t, \tau_0) = \text{long-term swelling (10}^{-6}\text{) at age } t, \text{ as measured from age } \tau_0
\]

\[
S_{w365} = \text{Swelling after 1 year.}
\]

\[
(t, \tau_0) = \text{Time since start of swelling (> 365 days)}
\]
PERDICTION OF DRYING SHRINKAGE & SWELLING

- In UK, BS8110:Part 2: 1985 given values of shrinkage and swelling after periods of exposure of 6 months and 30 yrs (see Fig.) for various relative humidities of storage and vol/surf. ratios.

- **Fig. 22** Prediction of shrinkage and swelling of high quality dense aggregate Concretes (from BS 8110: Part 2:1985).

- The data in the above Fig. apply to concrete made with high-quality, dense, non-shrinking aggregates and to concretes with an effective original water content of 8% of the original mass of concrete. (This value= 190lt/m3 of concrete).

- For concretes with other water contents, the shrinkage of above Fig. is adjusted in proportion to the actual water content.
**Fig. 22** Prediction of shrinkage and swelling of high quality dense aggregate Concretes (from BS 8110: Part 2:1985).
THERMAL MOVEMENT

• Concrete has positive coefficient of thermal expansion.
• Coefficient of thermal expansion is a function of
  – composition and
  – moisture condition
at the time of temperature change.

• Here, we are concerned with thermal movement caused by temperature changes within (-30 C, + 65C )

• Cement paste and aggregates have different thermal coefficients. The coefficient for concrete is affected by these two constituents.
THERMAL MOVEMENT

\[
\alpha_c = \alpha_p - \frac{2g (\alpha_p - \alpha_g)}{1 + \frac{k_p}{k_g} + g \left[1 - \frac{k_p}{k_g}\right]}
\]

(42)

g = volumetric content of aggregate.

\(k_p/k_g\) = stiffness ratio of cement paste to aggregate, approximately equal to the ratio of their moduli of elasticity.
• Typical values of the coefficient of thermal expansion of aggregate are given in Table 7.

• Values for cement paste vary between: $11 \times 10^{-6} - 20 \times 10^{-6}$ per C depending on the moisture condition.

• **Fig. 23** Influence of volume content of aggregate and of aggregate type on linear coefficient of thermal expansion of concrete, using the above equation ($\alpha_p = 15 \times 10^{-6}/C$)

• **Table 7** Coefficient of thermal expansion of 1:6 concretes made with different aggregates.
Fig. 23 Influence of volume content of aggregate and of aggregate type on linear coefficient of thermal expansion of concrete, using the above equation ($\alpha_p = 15 \times 10^{-6}/C$)
Table 7. Coefficient of thermal expansion of 1:6 concretes made with different aggregates

<table>
<thead>
<tr>
<th>Type of aggregate</th>
<th>Linear coefficient of thermal expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Air cured concrete $10^{-6}$ /C</td>
</tr>
<tr>
<td>Gravel</td>
<td>13.1</td>
</tr>
<tr>
<td>Granite</td>
<td>9.5</td>
</tr>
<tr>
<td>Quartzite</td>
<td>12.8</td>
</tr>
<tr>
<td>Dolorite</td>
<td>9.5</td>
</tr>
<tr>
<td>Sandstone</td>
<td>11.7</td>
</tr>
<tr>
<td>Limestone</td>
<td>7.4</td>
</tr>
<tr>
<td>Portland Stone</td>
<td>7.4</td>
</tr>
<tr>
<td>BF Slag</td>
<td>10.6</td>
</tr>
<tr>
<td>Foamed slag</td>
<td>12.1</td>
</tr>
</tbody>
</table>
THERMAL MOVEMENT

• Thermal coefficient of cement paste has two components;

True (kinetic) thermal coefficient:
• Caused by the molecular movement of the paste.

Hydrothermal expansion coefficient:
• Due to increase in the internal relative humidity as the temperature increases with a consequent expansion of the cement paste.

• No thermal expansion is possible if the paste is totally DRY or when it is SATURATED since there can be no increase in water vapor pressure.
Relation between ambient relative humidity and the linear coefficient of thermal expansion of neat cement paste cured normally.
Relation between the linear coefficient of thermal expansion and temperature of concrete specimens stored and tested at the age of 55 days under different humidity conditions.
There are 3 types of cracks:
1. Plastic cracks
2. Early age cracks (thermal)
3. Drying shrinkage cracks.

There are other types of non-structural cracks, listed in Table 8 and Fig. 25
Fig. 25 Schematic representation of the various types of cracking which can occur in concrete.
<table>
<thead>
<tr>
<th>Type of cracking</th>
<th>Symbol in Fig.25</th>
<th>Subdivision</th>
<th>Most common location</th>
<th>Primary case (excluding restraint)</th>
<th>Secondary causes/ factors</th>
<th>Remedy (assuming basic redesign is impossible); in all cases reduce restraint</th>
<th>Time of appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic settlement</td>
<td>A</td>
<td>Over reinforcement</td>
<td>Deep sections</td>
<td>Excess bleeding</td>
<td>Rapid early drying conditions</td>
<td>Reduce bleeding (air entrainment) or revibrate</td>
<td>10 min to 3 hr.</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Arching</td>
<td>Top of columns</td>
<td>Excess bleeding</td>
<td>Low rate of bleeding</td>
<td>Improve early curing</td>
<td>30 min to 6 hr.</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Change of depth</td>
<td>Trough and waffle slabs</td>
<td>Ditto plus steel near surface</td>
<td>Rapid cooling</td>
<td>Reduce heat and/or insulate</td>
<td>One day to two or three weeks</td>
</tr>
<tr>
<td>Plastic shrinkage</td>
<td>D</td>
<td>Diagonal</td>
<td>Roads and slabs</td>
<td>Rapid early drying</td>
<td>Low rate of bleeding</td>
<td>Improve early curing</td>
<td>30 min to 6 hr.</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>Random</td>
<td>Reinforced concrete slabs</td>
<td>Ditto plus steel near surface</td>
<td>Rapid cooling</td>
<td>Reduce heat and/or insulate</td>
<td>One day to two or three weeks</td>
</tr>
<tr>
<td></td>
<td>F</td>
<td>Over reinforcement</td>
<td>Reinforced concrete slabs</td>
<td>Ditto plus steel near surface</td>
<td>Rapid cooling</td>
<td>Reduce heat and/or insulate</td>
<td>One day to two or three weeks</td>
</tr>
<tr>
<td>Early thermal contraction</td>
<td>G</td>
<td>External restraint</td>
<td>Thick walls</td>
<td>Excess heat generation</td>
<td>Excess temperature gradients</td>
<td>Improve early curing</td>
<td>Several weeks or months</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>Internal restraint</td>
<td>Thick walls</td>
<td>Excess temperature gradients</td>
<td>Rapid cooling</td>
<td>Reduce water content Improve curing</td>
<td>Several weeks or months</td>
</tr>
<tr>
<td>Long-term drying shrinkage</td>
<td>I</td>
<td>Thin walls and slabs</td>
<td>Inefficient joints</td>
<td>Excess shrinkage Inefficient curing</td>
<td>Reduce water content Improve curing</td>
<td>Improve curing and finishing</td>
<td>One to seven days, sometimes much later</td>
</tr>
<tr>
<td>Crazing</td>
<td>J</td>
<td>Against formwork</td>
<td><code>Fair face</code> concrete</td>
<td>Impermeable formwork</td>
<td>Rich mixes Poor curing</td>
<td>Improve curing and finishing</td>
<td>One to seven days, sometimes much later</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>Floated concrete</td>
<td>Slabs</td>
<td>Over-trowelling</td>
<td>Poor curing</td>
<td>Improve curing and finishing</td>
<td>One to seven days, sometimes much later</td>
</tr>
<tr>
<td>Corrosion of reinforcing</td>
<td>L</td>
<td>Natural</td>
<td>Columns and beams</td>
<td>Lack of cover</td>
<td>Poor quality concrete</td>
<td>Eliminate causes listed</td>
<td>More than two years</td>
</tr>
<tr>
<td>Alkali-aggregate reaction</td>
<td>M</td>
<td>Damp locations</td>
<td>Reactive aggregate plus high-alkali cement</td>
<td>Eliminate causes</td>
<td></td>
<td>More than five years</td>
<td></td>
</tr>
</tbody>
</table>

Table 8. Classification of intrinsic cracks
TYPES OF CRACKING

- **Fig. 25** Schematic representation of the various types of cracking which can occur in concrete.

- **Plastic Cracks**: Develop before the concrete has hardened (between 1 to 8 hours after placing). They are in the form of plastic shrinkage cracks and plastic settlement cracks.

- Air entraining agents can be used to reduce the incidence of plastic settlement cracks (reduce bleeding) and also increase the cover to the top steel.

- **Drying shrinkage cracks**: In large sections they are induced by tensile stresses due to internal restraint caused by differential shrinkage between the surface and the interior of the concrete.

- Drying shrinkage cracks occur because of external restraint to movement provided by another part of the structure or by the sub-grade.

- Adequate curing is essential so as to increase the tensile strength of the concrete, together with the elimination of external restraints by the provision of **movement joints**.
• The width of shrinkage cracks can be controlled by placing reinforcement near to the surface as much as possible.

• Other types of cracking are caused by:
  – Corrosion of reinforcement
  – Alkali-aggregate reactions.

**Surface Crazing:**

Can be seen on walls or slabs.

They occur when the surface layer of concrete has a higher water content than the interior concrete.
SURFACE CRAZING