A 600-mm wide, 500-mm deep continuous footing carries a vertical downward load of 85 kN/m. The soil has $\gamma = 19 kN/m^3$. Using Boussinesq's method, compute $\Delta \sigma$, at a depth of 200 mm below the bottom of the footing at the following locations:

- Beneath the center of the footing
- 150 mm from the center of the footing
- 300 mm from the center of the footing (i.e., beneath the edge)
- 450 mm from the center of the footing

Plot the results in the form of a pressure diagram similar to those in Figure 5.10 in Chapter 5.

Hint: Use the principle of superposition.

A 3-ft square, 2-ft deep footing carries a column load of 28.2 k. An architect is proposing to build a new 4 ft wide, 2 ft deep continuous footing adjacent to this existing footing. The side of the new footing will be only 6 inches away from the side of the existing footing. The new footing will carry a load of 12.3 k/ft. $\gamma = 119 lb/ft^3$.

Develop a plot of $\Delta \sigma$, due to the new footing vs. depth along a vertical line beneath the center of the existing footing. This plot should extend from the bottom of the existing footing to a depth of 35 ft below the bottom of this footing.

Using the data from Problem 7.25, $C_r/(1 + e_o) = 0.08$ and $\gamma = 119 lb/ft^3$, compute the consolidation settlement of the old footing due to the construction and loading of the new footing. The soil is an overconsolidated (case I) silty clay, and the groundwater table is at a depth of 8 ft below the ground surface.

Using the SCHRIMPTANN.XLS spreadsheet and the subsurface data from Example 7.6, develop a plot of footing width, $B$, vs. column load, $P$, for square spread footings embedded 3 ft below the ground surface. Develop a $P$ vs. $B$ curve for each of the following settlements: 0.5 in, 1.0 in, and 1.5 in, and present all three curves on the same diagram.

Your greatest danger is letting the urgent things crowd out the important.

From Tyranny of the Urgent by Charles E. Hummel

This chapter shows how to use the results of bearing capacity and settlement computations, as well as other considerations, to develop spread footing designs that satisfy geotechnical requirements. These are the requirements that relate to the safe transfer of the applied loads from the footing to the ground. Chapter 9 builds on this information, and discusses the structural design aspects, which are those that relate to the structural integrity of the footing and the connection between the footing and the superstructure.

8.1 DESIGN FOR CONCENTRIC DOWNWARD LOADS

The primary load on most spread footings is the downward compressive load, $P$. This load produces a bearing pressure $q$ along the bottom of the footing, as described in Section 5.3. Usually we design such footings so that the applied load acts through the centroid (i.e., the column is located in the center of the footing). This way the bearing pressure is uniformly distributed along the base of the footing (or at least it can be assumed to be uniformly distributed) and the footing settles evenly.
Footing Depth

The depth of embedment, $D$, must be at least large enough to accommodate the required footing thickness, $T$, as shown in Figure 8.1. This depth is measured from the lowest adjacent ground surface to the bottom of the footing. In the case of footings overlain by a slab-on-grade floor, $D$ is measured from the subgrade below the slab.

Tables 8.1 and 8.2 present minimum $D$ values for various applied loads. These are the unfactored loads (i.e., the greatest from Equations 2.1–2.4). These $D$ values are intended to provide enough room for the required footing thickness, $T$. In some cases, a more detailed analysis may justify shallower depths, but $D$ should never be less than 300 mm (12 in). The required footing thickness, $T$, is governed by structural concerns, as discussed in Chapter 9.

**TABLE 8.1 MINIMUM DEPTH OF EMBEDMENT FOR SQUARE AND RECTANGULAR FOOTINGS**

<table>
<thead>
<tr>
<th>Load $P$ (kN/m)</th>
<th>Minimum $D$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–170</td>
<td>300</td>
</tr>
<tr>
<td>170–250</td>
<td>400</td>
</tr>
<tr>
<td>250–330</td>
<td>500</td>
</tr>
<tr>
<td>330–410</td>
<td>600</td>
</tr>
<tr>
<td>410–490</td>
<td>700</td>
</tr>
<tr>
<td>490–570</td>
<td>800</td>
</tr>
<tr>
<td>570–650</td>
<td>900</td>
</tr>
<tr>
<td>650–740</td>
<td>1000</td>
</tr>
</tbody>
</table>

**TABLE 8.2 MINIMUM DEPTH OF EMBEDMENT FOR CONTINUOUS FOOTINGS**

<table>
<thead>
<tr>
<th>Load $P$ (kN/m)</th>
<th>Minimum $D$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–170</td>
<td>300</td>
</tr>
<tr>
<td>170–250</td>
<td>400</td>
</tr>
<tr>
<td>250–330</td>
<td>500</td>
</tr>
<tr>
<td>330–410</td>
<td>600</td>
</tr>
<tr>
<td>410–490</td>
<td>700</td>
</tr>
<tr>
<td>490–570</td>
<td>800</td>
</tr>
<tr>
<td>570–650</td>
<td>900</td>
</tr>
<tr>
<td>650–740</td>
<td>1000</td>
</tr>
</tbody>
</table>

Sometimes it is necessary to use embedment depths greater than those listed in Tables 8.1 and 8.2. This situations include the following:

- The upper soils are loose or weak, or perhaps consist of a fill of unknown quality. In such cases, we usually extend the footing through these soils and into the underlying competent soils.
- The soils are prone to frost heave, as discussed later in this section. The customary procedure in such soils is to extend the footings to a depth that exceeds the depth of frost penetration.
- The soils are expansive. One of the methods of dealing with expansive soils is to extend the footings to a greater depth. This gives them additional flexural strength, and places them below the zone of greatest moisture fluctuation. Chapter 19 discusses this technique in more detail.
- The soils are prone to scour, which is erosion caused by flowing water. Footings in such soils must extend below the potential scour depth. This is discussed in more detail later in this chapter.
- The footing is located near the top of a slope in which there is some, even remote, possibility of a shallow landslide. Such footings should be placed deeper than usual in order to provide additional protection against undermining from any such slides.

Sometimes we also may need to specify a maximum depth. It might be governed by such considerations as:

- Potential undermining of existing foundations, structures, streets, utility lines, etc.
- The presence of soft layers beneath harder and stronger near-surface soils, and the desire to support the footings in the upper stratum.
Chapter 8 Spread Footings—Geotechnical Design

- A desire to avoid working below the groundwater table, and thus avoid construction dewatering expenses.
- A desire to avoid the expense of excavation shoring, which may be needed for footing excavations that are more than 1.5 m (5 ft) deep.

Footing Width

Sometimes bearing capacity and settlement concerns can be addressed by increasing the footing depth. For example, if the near-surface soils are poor, but those at slightly greater depths are substantially better, bearing capacity and settlement problems might be solved by simply deepening the footing until it reaches the higher quality stratum. However, in more uniform soil profiles, we usually satisfy bearing capacity and settlement requirements by adjusting the footing width, B. Increasing B causes the bearing pressure, \( q \), to decrease, which improves the factor of safety against a bearing capacity failure and decreases the settlement.

Most structures require many spread footings, perhaps dozens of them, so it is inconvenient to perform custom bearing capacity and settlement analyses for each one. Instead, geotechnical engineers develop generic design criteria that are applicable to the entire site, then the structural engineer sizes each footing based on its load and these generic criteria. We will discuss two methods of presenting these design criteria: the allowable bearing pressure method and the design chart method.

Allowable Bearing Pressure Method

The allowable bearing pressure, \( q_A \), is the largest bearing pressure that satisfies both bearing capacity and settlement criteria. In other words, it is equal to the allowable bearing capacity, \( q_A \), or the \( q \) that produces the greatest acceptable settlement, whichever is less. Normally we develop a single \( q_A \) value that applies to the entire site, or at least to all the footings of a particular shape at that site.

Geotechnical engineers develop \( q_A \) using the following procedure:

1. Select a depth of embedment, \( D \), as described earlier in this chapter. If different depths of embedment are required for various footings, perform the following computations using the smallest \( D \).
2. Determine the design groundwater depth, \( D_e \). This should be the shallowest groundwater depth expected to occur during the life of the structure.
3. Determine the required factor of safety against a bearing capacity failure (see Figure 6.11).
4. Using the techniques described in Chapter 6, perform a bearing capacity analysis on the footing with the smallest applied normal load. This analysis is most easily performed using the BEARING.XLS spreadsheet. Alternatively, it may be performed as follows:

\[
q = \frac{P + W}{A} - u_0 \leq q_A
\]  

8.1 Design for Concentric Downward Loads

a. Using Equation 5.1 or 5.2, write the bearing pressure, \( q \), as a function of \( B \).

b. Using Equation 6.4, 6.5, 6.6, or 6.13, along with Equation 6.36, write the allowable bearing capacity, \( q_A \), as a function of \( B \).

c. Set \( q = q_A \) and solve for \( B \).

d. Using Equation 6.4, 6.5, 6.6, or 6.13, along with Equation 6.36 and the \( B \) from Step c, determine the allowable bearing capacity, \( q_A \).

5. Using the techniques described in Chapter 2, determine the allowable total and differential settlements, \( \delta_t \) and \( \delta_d \). Normally the structural engineer performs this step and provides these values to the geotechnical engineer.

6. Using local experience or Table 7.5, select an appropriate \( \delta_t / \delta_d \) ratio.

7. If \( \delta_d \geq \delta_t (\delta_t / \delta_d) \), then designing the footings to satisfy the total settlement requirement (\( \delta_t \leq \delta_t \)) will implicitly satisfy the differential settlement requirement as well (\( \delta_d \leq \delta_d \)). Therefore, continue to Step 8 using \( \delta_t \). However, if \( \delta_d < \delta_t / \delta_d \), it is necessary to reduce \( \delta_t \) to keep differential settlement under control (see Example 7.8). In that case, continue to Step 8 using a revised \( \delta_t = \delta_t / (\delta_t / \delta_d) \).

8. Using the \( \delta_t \) value obtained from Step 7, and the techniques described in Chapter 7, perform a settlement analysis on the footing with the largest applied normal load. This analysis is most easily performed using the SETTLEMENT.XLS or SCHMERTMANN.XLS spreadsheets. Determine the maximum bearing pressure, \( q \), that keeps the total settlement within tolerable limits (i.e., \( \delta \leq \delta_t \)).

9. Set the allowable bearing pressure, \( q_A \), equal to the lower of the \( q_A \) from Step 4 or \( q_A \) from Step 8. Express it as a multiple of 500 lb/ft² or 25 kPa.

If the structure will include both square and continuous footings, we can develop separate \( q_A \) values for each.

We use the most lightly loaded footing for the bearing capacity analysis because it is the one that has the smallest \( B \) and therefore the lowest ultimate bearing capacity (per Equations 6.4-6.6). Thus, this footing has the lowest \( q_A \) of any on the site, and it is conservative to design the other footings using this value.

However, we use the most heavily loaded footing (i.e., the one with the largest \( B \)) for the settlement analysis, because it is the one that requires the lowest value of \( q \) to satisfy settlement criteria. To understand why this is so, compare the two footing in Figure 8.2. Both of these footings have the same bearing pressure, \( q \). However, since a greater volume of soil is being stressed by the larger footing, it will settle more than the smaller footing. For footings on clays loaded to the same \( q \), the settlement is approximately proportional to \( B \), while in sands it is approximately proportional to \( B^{1/3} \). Therefore, the larger footing is the more critical one for settlement analyses.

The geotechnical engineer presents \( q_A \), along with other design criteria, in a written report. The structural engineer receives this report, and uses the recommended \( q_A \) to design the spread footings such that \( q \leq q_A \). Thus, for square, rectangular, and circular footings:

\[
q = \frac{P + W}{A} - u_0 \leq q_A
\]  

(8.1)
For continuous footings:

\[ A = \frac{P}{q} \]

Where:

\( A \) = required base area

For square footings, \( A = B^2 \)

For rectangular footings, \( A = BL \)

For circular footings, \( A = \pi B^2/4 \)

\( B \) = footing width or diameter

\( L \) = footing length

\( P \) = applied normal load (unfactored)

\( P/b \) = applied normal load per unit length (unfactored)
Chapter 8 Spread Footings—Geotechnical Design

8.1 Design for Concentric Downward Loads

The equivalent modulus of the proposed compacted fill is:

\[ E = E_v \sqrt{OCR} + \beta \cdot N_60 \]

\[ = 7000 \sqrt{3} + (16,000)(60) \]

\[ = 1,100,000 \text{ lb/ft}^2 \]

Based on this data, we can perform the settlement analysis using the following equivalent modulus values:

<table>
<thead>
<tr>
<th>Depth (ft)</th>
<th>( E ) (lb/ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10</td>
<td>1,100,000</td>
</tr>
<tr>
<td>10 - 20</td>
<td>1,000,000</td>
</tr>
<tr>
<td>&gt; 20</td>
<td>1,700,000</td>
</tr>
</tbody>
</table>

Using the SCHMERTMANN.XLS spreadsheet with \( P = 900 \text{ k} \) and \( \beta = 1.0 \) in produces \( q = 6,700 \text{ lb/ft}^2 \)

Step 9 — \( 6,700 < 10,500 \), so settlement controls the design. Rounding to a multiple of 500 \text{ lb/ft}^2 \) gives:

\[ q_A = 6500 \text{ lb/ft}^2 \]

Design Chart Method

The allowable bearing pressure method is sufficient for most small to medium-size structures. However, larger structures, especially those with a wide range of column loads, warrant a more precise method: the design chart. This added precision helps us reduce both differential settlements and construction costs.
Instead of using a single allowable bearing pressure for all footings, it is better to use a higher pressure for small ones and a lower pressure for large ones. This method reduces the differential settlements and avoids the material waste generated by the allowable bearing pressure method. This concept is implicit in a design chart such as the one in Figure 8.4.

Use the following procedure to develop design charts:

1. Determine the footing shape (i.e., square, continuous, etc.) for this design chart. If different shapes are to be used, each must have its own design chart.
2. Select the depth of embedment, D, using the guidelines described earlier in this chapter. If different D values are required for different footings, perform these computations using the smallest D.
3. Determine the design groundwater depth, \( D'' \). This should be the shallowest groundwater depth expected to occur during the life of the structure.
4. Select the design factor of safety against a bearing capacity failure (see Figure 6.11).
5. Set the footing width \( B \) equal to 300 mm or 1 ft, then conduct a bearing capacity analysis and compute the column load that corresponds to the desired factor of safety. Plot this \( (B, P) \) data point on the design chart. Then select a series of new \( B \) values, compute the corresponding \( P \), and plot the data points. Continue this process until the computed \( P \) is slightly larger than the maximum design column load. Finally, connect these data points with a curve labeled “bearing capacity.” The spreadsheet developed in Chapter 6 makes this task much easier.
6. Develop the first settlement curve as follows:
   a. Select a settlement value for the first curve (e.g., 0.25 in).
   b. Select a footing width, \( B \), that is within the range of interest and arbitrarily select a corresponding column load, \( P \). Then, compute the settlement of this footing using the spreadsheets developed in Chapters 6 and 7, or some other suitable method.
   c. By trial-and-error, adjust the column load until the computed settlement matches the value assigned in step a. Then, plot the point \( B, P \) on the design chart.
   d. Repeat steps b and c with new values of \( B \) until a satisfactory settlement curve has been produced.
7. Repeat step 6 for other settlement values, thus producing a family of settlement curves on the design chart. These curves should encompass a range of column loads and footing widths appropriate for the proposed structure.
8. Using the factors in Table 7.5, develop a note for the design chart indicating the design differential settlements are \( \% \) of the total settlements.

Once the design chart has been obtained, the geotechnical engineer gives it to the structural engineer who sizes each footing using the following procedure:

1. Compute the design load, \( P \), which is the largest load computed from Equations 2.1, 2.2, 2.3a, or 2.4a. Note that this is the unfactored load, even if the superstructure has been designed using the factored load.
2. Using the bearing capacity curve on the design chart, determine the minimum required footing width, \( B \), to support the load \( P \) while satisfying bearing capacity requirements.
3. Using the settlement curve that corresponds to the allowable total settlement, \( \delta_s \), determine the footing width, \( B \), that corresponds to the design load, \( P \). This is the minimum width required to satisfy total settlement requirements.
4. Using the \( \delta_B/\delta \) ratio stated on the design chart, compute the differential settlement, \( \delta_D \), and compare it to the allowable differential settlement, \( \delta_{Da} \).
5. If the differential settlement is excessive (\( \delta_D > \delta_{Da} \)), then use the following procedure:
   a. Use the allowable differential settlement, \( \delta_{Da} \), and the \( \delta_B/\delta \) ratio to compute a new value for allowable total settlement, \( \delta_s \). This value implicitly satisfies both total and differential settlement requirements.
b. Using the settlement curve on the design chart that corresponds to this revised $\delta_w$ determine the required footing width, $B$. This footing width is smaller than that computed in step 3, and satisfies both total and differential settlement criteria.

6. Select the larger of the $B$ values obtained from the bearing capacity analysis (step 2) and the settlement analysis (step 3 or 5b). This is the design footing width.

7. Repeat steps 1 to 6 for the remaining columns.

These charts clearly demonstrate how the bearing capacity governs the design of narrow footings, whereas settlement governs the design of wide ones.

The advantages of this method over the allowable bearing pressure method include:

- The differential settlements are reduced because the bearing pressure varies with the footing width.
- The selection of design values for total and differential settlement becomes the direct responsibility of the structural engineer, as it should be. (With the allowable bearing pressure method, the structural engineer must give allowable settlement data to the geotechnical engineer who incorporates it into $q_A$.)
- The plot shows the load-settlement behavior, which we could use in a soil-structure interaction analysis.

Example 8.2

Develop a design chart for the proposed arena described in Example 8.1, then use this chart to determine the required width for a footing that is to support a 300-k column load.

Solution

Bearing capacity analyses (based on BEARING.XLS spreadsheet)

<table>
<thead>
<tr>
<th>$B$ (ft)</th>
<th>$P$ (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>74</td>
</tr>
<tr>
<td>4</td>
<td>146</td>
</tr>
<tr>
<td>5</td>
<td>251</td>
</tr>
<tr>
<td>6</td>
<td>395</td>
</tr>
<tr>
<td>7</td>
<td>582</td>
</tr>
<tr>
<td>8</td>
<td>818</td>
</tr>
<tr>
<td>9</td>
<td>1109</td>
</tr>
</tbody>
</table>

Settlement analyses (based on SCHMERTMANN.XLS spreadsheet)

Column loads to obtain a specified total settlement

<table>
<thead>
<tr>
<th>$\delta$ (in)</th>
<th>$B = 2$ ft</th>
<th>$B = 7$ ft</th>
<th>$B = 12$ ft</th>
<th>$B = 17$ ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>32 k</td>
<td>145 k</td>
<td>280 k</td>
<td>450 k</td>
</tr>
<tr>
<td>0.50</td>
<td>56 k</td>
<td>260 k</td>
<td>510 k</td>
<td>830 k</td>
</tr>
<tr>
<td>0.75</td>
<td>77 k</td>
<td>365 k</td>
<td>720 k</td>
<td>1190 k</td>
</tr>
<tr>
<td>1.00</td>
<td>97 k</td>
<td>465 k</td>
<td>925 k</td>
<td>1725 k</td>
</tr>
<tr>
<td>1.25</td>
<td>115 k</td>
<td>555 k</td>
<td>1120 k</td>
<td>1920 k</td>
</tr>
</tbody>
</table>

The result of these analyses are plotted in Figure 8.5.

According to this design chart, a 300-k column load may be supported on a 5 ft. 6 in wide footing. This is much smaller than the 7 ft. 0 in wide footing in Example 8.1.

Use $B = 5$ ft 6 in $\Rightarrow$ Answer

QUESTIONS AND PRACTICE PROBLEMS

8.1 Which method of expressing footing width criteria (allowable bearing pressure or design chart) would be most appropriate for each of the following structures?
Chapter 8 Spread Footings—Geotechnical Design

8.2 Design for Eccentric or Moment Loads

8.9 Several cone penetration tests have been conducted in a young, normally consolidated silica sand. Based on these tests, an engineer has developed the following design soil profile:

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>q (kg/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2.0</td>
<td>40</td>
</tr>
<tr>
<td>2.0-3.5</td>
<td>78</td>
</tr>
<tr>
<td>3.5-4.0</td>
<td>125</td>
</tr>
<tr>
<td>4.0-6.5</td>
<td>100</td>
</tr>
</tbody>
</table>

This soil has an average unit weight of 18.1 kN/m³ above the groundwater table and 20.8 kN/m³ below. The groundwater table is at a depth of 3.1 m.

Using this data with the spreadsheets described in Chapters 6 and 7, create a design chart for 1.0-m deep square footings. Consider footing widths of up to 4 m and column loads up to 1500 kN. A factor of safety of 2.5, and a design life of 50 years.

8.2 Explain why an 8-ft wide footing with \( q = 3000 \text{ lb/ft}^2 \) will settle more than a 3-ft wide one with the same \( q \).

8.3 Under what circumstances would bearing capacity most likely control the design of spread footings? Under what circumstances would settlement usually control?

8.4 A proposed building will have column loads ranging from 40 to 300 kN. All of these columns will be supported on square spread footings. When computing the allowable bearing pressure, \( q_{as} \), which load should be used to perform the bearing capacity analyses? Which should be used to perform the settlement analyses?

8.5 A proposed building will have column loads ranging from 50 to 250 kN. These columns are to be supported on spread footings which will be founded in a silty sand with the following engineering properties: \( \gamma = 119 \text{ lb/ft}^3 \) above the groundwater table and 122 lb/ft³ below. \( c' = 0 \), \( \phi' = 32^\circ \), \( N_{60} = 30 \). The groundwater table is 15 ft below the ground surface. The required factor of safety against a bearing capacity failure must be at least 2.5 and the allowable settlement, \( s_a \), is 0.75 in.

Compute the allowable bearing pressure for square spread footings founded 2 ft below the ground surface at this site. You may use the spreadsheets described in Chapters 6 and 7 to perform the computations, or you may do so by hand. Then, comment on the feasibility of using spread footings at this site.

8.6 A proposed office building will have column loads between 200 and 1000 kN. These columns are to be supported on spread footings which will be founded in a silt clay with the following engineering properties: \( \gamma = 15.1 \text{ kN/m}^3 \) above the groundwater table and 16.5 kN/m³ below, \( s_a = 200 \text{ kPa} \), \( C/(1+e_0) = 0.020 \), \( \sigma_{f}^{*} = 400 \text{ kPa} \). The groundwater table is 5 m below the ground surface. The required factor of safety against a bearing capacity failure must be at least 3 and the allowable settlement, \( s_a \), is 20 mm.

Compute the allowable bearing pressure for square spread footings founded 0.5 m below the ground surface at this site. You may use the spreadsheets described in Chapters 6 and 7 to perform the computations, or you may do so by hand. Then, comment on the feasibility of using spread footings at this site.

8.7 A series of columns carrying vertical loads of 20 to 90 k are to be supported on 3-ft deep square footings. The soil below is a clay with the following engineering properties: \( \gamma = 105 \text{ lb/ft}^3 \) above the groundwater table and 110 lb/ft³ below, \( s_a = 3000 \text{ lb/ft}^3 \), \( C/(1+e_0) = 0.03 \) in the upper 10 ft and 0.05 below. Both soil strata are overconsolidated Case I. The groundwater table is 5 ft below the ground surface. The factor of safety against a bearing capacity failure must be at least 3. Use the spreadsheets described in Chapters 6 and 7 to compute the allowable bearing pressure, \( q_{as} \). The allowable settlement is 1.4 in.

8.8 Using the information in Problem 8.7, develop a design chart. Consider footing widths of up to 12 ft.

8.2 Design for Eccentric or Moment Loads

Sometimes it becomes necessary to build a footing in which the downward load, \( P \), does not act through the centroid, as shown in Figure 8.6a. One example is in an exterior footing in a structure located close to the property line, as shown in Figure 5.2. The bearing pressure beneath such footings is skewed, as discussed in Section 5.3.
8.2 Design for Eccentric or Moment Loads

b. Compute the effective footing dimensions:

\[ B' = B - 2e_B \]
\[ L' = L - 2e_L \]

(8.4)
(8.5)

This produces an equivalent footing with an area \( A' = B' \times L' \) as shown in Figure 8.7.

4. Compute the equivalent bearing pressure using:

\[ q_{\text{equiv}} = \frac{P + W}{B'L'} - u_0 \]

(8.6)

5. Compare \( q_{\text{equiv}} \) with the allowable bearing pressure, \( q_A \). If \( q_{\text{equiv}} \leq q_A \), then the design is satisfactory. If not, then increase the footing size as necessary to satisfy this criterion.

Example 8.3

A 5-ft square, 2-ft deep footing supports a vertical load of 80 k and a moment load of 60 ft-k. The underlying soil has an allowable bearing pressure, \( q_A \), of 3500 Ib/ft² and the groundwater table is at a great depth. Is this design satisfactory?

Solution

Using Equation 5.5:

\[ W_f = (5 \text{ ft})(5 \text{ ft})(2 \text{ ft})(150 \text{ lb/ft}^3) = 7500 \text{ lb} \]

Use the following process to design for footings with eccentric or moment loads:

1. Develop preliminary values for the plan dimensions \( B \) and \( L \). If the footing is square, then \( B = L \). These values might be based on a concentric downward load analysis, as discussed in Section 8.1, or on some other method.

2. Determine if the resultant of the bearing pressure acts within the middle third of the footing (for one-way loading) or within the kern (for two-way loading). The tests for these conditions are described in Equations 5.9 and 5.10, and illustrated in Examples 5.4 and 5.5. If these criteria are not satisfied, then some of the footing will tend to lift off the soil, which is unacceptable. Therefore, any such footings need to be modified by increasing the width or length, as illustrated in Example 5.5.

3. Using the following procedure, determine the effective footing dimensions, \( B' \) and \( L' \), as shown in Figure 8.7 (Meyerhof, 1963; Brinch Hansen, 1970):

a. Using Equations 5.3 to 5.6, compute the bearing pressure eccentricity in the \( B \) and/or \( L \) directions (\( e_B, e_L \)).

Another, more common possibility is a footing that is subjected to an applied moment load, \( M \), as shown in Figure 8.6b. This moment may be permanent, but more often it is a temporary load due to wind or seismic forces acting on the structure. These moment loads also produce a skewed bearing pressure.

Use the following process to design for footings with eccentric or moment loads:

1. Develop preliminary values for the plan dimensions \( B \) and \( L \). If the footing is square, then \( B = L \). These values might be based on a concentric downward load analysis, as discussed in Section 8.1, or on some other method.

2. Determine if the resultant of the bearing pressure acts within the middle third of the footing (for one-way loading) or within the kern (for two-way loading). The tests for these conditions are described in Equations 5.9 and 5.10, and illustrated in Examples 5.4 and 5.5. If these criteria are not satisfied, then some of the footing will tend to lift off the soil, which is unacceptable. Therefore, any such footings need to be modified by increasing the width or length, as illustrated in Example 5.5.

3. Using the following procedure, determine the effective footing dimensions, \( B' \) and \( L' \), as shown in Figure 8.7 (Meyerhof, 1963; Brinch Hansen, 1970):

   a. Using Equations 5.3 to 5.6, compute the bearing pressure eccentricity in the \( B \) and/or \( L \) directions (\( e_B, e_L \)).

   \[ e = \frac{M}{P + W} \quad \text{where} \quad \frac{M}{80 \text{ k} + 7.5 \text{ k}} = 0.686 \text{ ft} \]

   \[ \frac{B}{6} = 5 \text{ ft} \quad \text{and} \quad \frac{L}{6} = 0.833 \text{ ft} \]

   \[ e = \frac{B}{6} \quad \text{OK for eccentric loading} \]

   \[ B' = B - 2e_B = 5 - (2)(0.686) = 3.63 \text{ ft} \]

   \[ q_{\text{equiv}} = \frac{P + W}{A} - u_0 = \frac{80,000 \text{ lb} + 7,500 \text{ lb}}{(3.63 \text{ ft})(6 \text{ ft})} - 0 = 4821 \text{ lb/ft}^2 \]

Since \( q_{\text{equiv}} > q_A \) (4821 > 3500), this design is not satisfactory. This is true even though eccentric loading requirement \( (e \leq B/6) \) has been met. Therefore, a larger \( B \) is required. \( \Rightarrow \text{Answer} \)

Further trials will demonstrate that \( B = 6 \text{ ft} \) in satisfies all of the design criteria.
### 8.3 DESIGN FOR SHEAR LOADS

Some footings are also subjected to applied shear loads, as shown in Figure 8.8. These loads may be permanent, as those from retaining walls, or temporary, as with wind or seismic loads on buildings.

Shear loads are resisted by passive pressure acting on the side of the footing, and by sliding friction along the bottom. The allowable shear capacity, \( V_a \), for footings located above the groundwater table at a site with a level ground surface is:

\[
V_a = \left( P + W_f \right) \mu_a + P_p - P_u
\]  
(8.7)

Passive and active forces are discussed in Chapter 23. However, rather than individually computing them for each footing, it is usually easier to compute \( \lambda \), which is the net result of the active and passive pressures expressed in terms of an equivalent fluid density. In other words, we evaluate the problem as if the soil along one side of the footing is replaced with a fluid that has a unit weight of \( \lambda \), then using the principles of fluid statics to compute the equivalent of \( P_p - P_u \). Thus, for square footings, Equation 8.7 may be rewritten as:

\[
V_a = \left( P + W_f \right) \mu_a + 0.5 \lambda \beta D^2
\]  
(8.8)

\[
\mu_a = \frac{\mu}{F}
\]  
(8.9)

\[
\lambda = \frac{\gamma \left[ \tan^2 \left( 45^\circ + \phi/2 \right) - \tan^2 \left( 45^\circ - \phi/2 \right) \right]}{F}
\]  
(8.10)

Equation 8.10 considers only the frictional strength of the soil. In some cases, it may be appropriate to also consider the cohesive strength using the techniques described in Chapter 23.

When quoting or using \( \mu \) and \( \lambda \), it is important to clearly indicate whether they are ultimate and allowable values. This is often a source of confusion, which can result in compounding factors of safety, or designing without a factor of safety. Normally, geotechnical engineering reports quote allowable values of these parameters.

The engineer also must be careful to use the proper value of \( P \) in Equation 8.8. Typically, multiple load combinations must be considered, and the shear capacity must be satisfactory for each combination. Thus, the \( P \) for a particular analysis must be the minimum necessary.

#### TABLE 8.3 DESIGN VALUES OF \( \mu \) FOR CAST-IN-PLACE CONCRETE

(U.S. Navy, 1982b)

<table>
<thead>
<tr>
<th>Soil or Rock Classification</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean sound rock</td>
<td>0.70</td>
</tr>
<tr>
<td>Clean gravel, gravel-sand mixtures, coarse sand</td>
<td>0.65-0.60</td>
</tr>
<tr>
<td>Clean fine-to-medium sand, silty medium-to-coarse sand, silty or clayey gravel</td>
<td>0.45-0.55</td>
</tr>
<tr>
<td>Clean fine sand, silty or clayey fine to medium sand</td>
<td>0.35-0.45</td>
</tr>
<tr>
<td>Fine sandy silt, nonplastic silt</td>
<td>0.30-0.35</td>
</tr>
<tr>
<td>Very stiff and hard residual or overconsolidated clay</td>
<td>0.40-0.50</td>
</tr>
<tr>
<td>Medium stiff and stiff clay and silty clay</td>
<td>0.30-0.35</td>
</tr>
</tbody>
</table>
normal load that would be present when the design shear load acts on the footing. For example, if $V$ is due to wind loads on a building, $P$ should be based on dead load only because the live load might not be present when the wind loads occur. If the wind load also causes an upward normal load on the footing, then $P$ would be equal to the dead load minus the upward wind load.

**Example 8.4**

A $6\text{ ft} \times 6\text{ ft} \times 2.5\text{ ft}$ deep footing supports a column with the following design loads: $P = 112\text{ kN}$, $V = 20\text{ kN}$. The soil is a silty fine-to-medium sand with $f' = 29\text{ kPa}$, and the groundwater table is well below the bottom of the footing. Check the shear capacity of this footing and determine if the design will safely withstand the design shear load.

**Solution**

Per Table 8.3: $\mu = 0.35-0.45$

Per Equation 8.11: $\mu = \tan \left[0.7(29)\right] = 0.37$

Use $\mu = 0.38$

$\lambda_V = \frac{\mu}{F} = \frac{0.38}{1.5} = 0.25$

$W_P = \frac{120[\tan^2(45 + 29/2) - \tan^2(45 - 29/2)]}{2} = 152\text{ lb/ft}^2$

$W_f = \frac{(6)(2.5)(150)}{1000} = 13,500\text{ lb}$

$V_P = (112 + 12.5)(0.25) + (0.5)(\frac{152}{1000})(6)(2.5) = 34\text{ k}

$V < V_\lambda(20 < 34)$ so the footing has sufficient lateral load capacity

Footings subjected to applied shear loads also have a smaller ultimate bearing capacity, which may be assessed using the factors in Vesic’s method, as described in Chapter 6. This reduction in bearing capacity is often ignored when the shear load is small (i.e., less than about 0.20 $P$), but it can become significant with larger shear loads.

### QUESTIONS AND PRACTICE PROBLEMS

**8.10** A square spread footing with $B = 1000\text{ mm}$ and $D = 500\text{ mm}$ supports a column with the following design loads: $P = 150\text{ kN}$, $M = 22\text{ kN-m}$. The underlying soil has an allowable bearing pressure of 200 kPa. Is this design acceptable? If not, compute the minimum required footing width and express it as a multiple of 100 mm.

**8.11** A $3\text{ ft} \times 7\text{ ft}$ rectangular footing is to be embedded 2 ft into the ground and will support a single centrally-located column with the following design loads: $P = 50\text{ kN}$, $M = 80\text{ ft-k}$ (acts in long direction only). The underlying soil is a silty sand with $c' = 0$, $\phi' = 31^\circ$, $\gamma = 123\text{ lb/ft}^2$. and a very deep groundwater table. Using a factor of safety of 2.5, determine if this design is acceptable for bearing capacity.

**8.12** A $4\text{ ft} \times 4\text{ ft}$ square spread footing embedded 1.5 ft into the ground is subjected to the following design loads: $P = 25\text{ kN}$, $V = 6\text{ kN}$. The underlying soil is a well-graded sand with $c' = 0$, $\phi' = 36^\circ$, $\gamma = 120\text{ lb/ft}^3$, and a very deep groundwater table. Using a factor of safety of 2.5 on bearing capacity, 2 on passive pressure, and 1.5 on sliding friction, determine if this design is acceptable for bearing capacity and for lateral load capacity.
Most building codes allow this one-third increase for short term loads [ICBO 1612.3.1805.2, and Table 18-I-A], [BOCA 1805.2], [ICC 1605.3.2 and Table 1804.2]. In addition, most building codes permit the geotechnical engineer to specify allowable bearing pressures based on a geotechnical investigation, and implicitly allow the flexibility to express separate allowable bearing pressures for short- and long-term loading conditions.

This one-third increase is appropriate for most soil conditions. However, it probably should not be used for foundations supported on soft clayey soils, because they may have lower strength when subjected to strong wind or seismic loading (Krinitzky, et al., 1993). In these soils, the foundations should be sized using a design load equal to the greatest of Equations 2.1 to 2.4 and the $q_A$ value from Chapter 8.

There are two ways to implement this one-third increase in the design process for downward loads:

**Method 1:**
1. Compute the long duration load as the greater of that produced by Equations 2.1 and 2.2.
2. Size the foundation using the load from Step 1, the $q_A$ from Chapter 8, and Equation 8.2 or 8.3.
3. Compute the short duration load as the greater of that produced by Equations 2.3 and 2.4.
4. Size the foundation using the load from Step 3, 1.33 times the $q_A$ from Chapter 8, and Equation 8.2 or 8.3.
5. Use the larger of the footing sizes from Steps 2 and 4 (i.e., the final design may be controlled by either the long term loading condition or the short term loading condition).

This method is a straightforward application of the principle described above, but can be tedious to implement. The second method is an attempt to simplify the analysis while producing the same design:

**Method 2:**
1. Compute the design load as the greatest of that produced by Equations 2.1, 2.2, 2.3a, and 2.4a.
2. Size the foundation using the load from Step 1, the $q_A$ from Chapter 8, and Equation 8.2 or 8.3.

Therefore, the author recommends using Method 2.

The design process for shear loads also may use either of these two methods. Once again, it is often easier to use Method 2.

**Special Seismic Considerations**

Loose sandy soils pose special problems when subjected to seismic loads, especially if these soils are saturated. The most dramatic problem is soil liquefaction, which is the sudden loss of shear strength due to the build-up of excess pore water pressures (see Coduto, 1999). This loss in strength can produce a bearing capacity failure, as shown in Figure 8.9. Another problem with loose sands, even if they are not saturated and not prone to liquefaction, is earthquake-induced settlement. In some cases, such settlements can be very large.

Earthquakes also can induce landslides, which can undermine foundations built near the top of a slope. This type of failure occurred in Anchorage, Alaska, during the 1964 earthquake, as well as elsewhere. The evaluation of such problems is a slope stability concern, and thus is beyond the scope of this book.

**8.5 LIGHTLY-LOADED FOOTINGS**

The principles of bearing capacity and settlement apply to all sizes of spread footings. However, the design process can be simplified for lightly-loaded footings. For purposes of geotechnical foundation design, we will define lightly-loaded footings as those subjected to vertical loads less than 200 kN (45 k) or 60 kN/m (4 k/ft). These include typical one- and two-story wood-frame buildings, and other similar structures. The foundations for such structures are small, and do not impose large loads onto the ground, so extensive
Presumptive Allowable Bearing Pressures

Spread footings for lightweight structures are often designed using presumptive allowable bearing pressures (also known as prescriptive bearing pressures) which are allowable bearing pressures obtained directly from the soil classification. These presumptive bearing pressures appear in building codes, as shown in Table 8.4. They are easy to implement, and do not require borings, laboratory testing, or extensive analyses. The engineer simply obtains the \( q_\lambda \) value from the table and uses it with Equation 8.2 or 8.3 to design the footings.

### TABLE 8.4 PRESUMPTIVE ALLOWABLE BEARING PRESSURES FROM VARIOUS BUILDING CODES

<table>
<thead>
<tr>
<th>Soil or Rock Classification</th>
<th>Allowable Bearing Pressure, ( q_\lambda ) lb/ft(^2) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sedimentary rock</td>
<td>4,000–12,000</td>
</tr>
<tr>
<td>Sandy gravel or gravel</td>
<td>2,000–6,000</td>
</tr>
<tr>
<td>Sand, silty sand, clayey sand, silty gravel, or clayey gravel</td>
<td>1,500–4,500</td>
</tr>
<tr>
<td>Clay, sandy clay, silty clay, or clayey silt</td>
<td>1,000–3,000</td>
</tr>
</tbody>
</table>


*The values in this table are for illustrative purposes only and are not a complete description of the code provisions. Portions of the table include the author’s interpretations to classify the presumptive bearing values into uniform soil groups. Refer to the individual codes for more details.

Presumptive allowable bearing pressures have been used since the late nineteenth century, and thus predate bearing capacity and settlement analyses. Today they are used primarily for lightweight structures on sites known to be underlain by good soils. Although presumptive bearing pressures are usually conservative (i.e., they produce larger footings), the additional construction costs are small compared to the savings in soil exploration and testing costs.

However, it is inappropriate to use presumptive bearing pressures for larger structures founded on soil because they are not sufficiently reliable. Such structures warrant more extensive engineering and design, including soil exploration and testing. They also should not be used on sites underlain by poor soils.

**Minimum Dimensions**

If the applied loads are small, such as with most one- or two-story wood-frame structures, bearing capacity and settlement analyses may suggest that extremely small footings would be sufficient. However, from a practical perspective, very small footings are not acceptable for the following reasons:

- Construction of the footing and the portions of the structure that connect to it would be difficult.
- Excavation of soil to build a footing is by no means a precise operation. If the footings are small, the ratio of the construction tolerances to the footing dimensions would be large, which would create other construction problems.
- A certain amount of flexural strength is necessary to accommodate nonuniformities in the loads and local inconsistencies in the soil, but an undersized footing would have little flexural strength.

Therefore, all spread footings should be built with certain minimum dimensions. Figure 8.10 shows typical minimums. In addition, building codes sometimes dictate other minimums. For example, the Uniform Building Code and the International Building Code stipulate certain minimum dimensions for footings that support wood-frame structures. The minimum dimensions for continuous footings are presented in Table 8.5, and those for square footings are presented in Note 3 of the table. This code also allows the geotechnical engineer to supersede these minimum dimensions [UBC 1806.1, IBC 1805.21].

**Potential Problems**

Although the design of spread footings for lightweight structures can be a simple process, as just described, be aware that such structures are not immune to foundation problems. Simply following these presumptive bearing pressures and code minimums does not necessarily produce a good design. Engineers need to know when these simple design guidelines are sufficient, and when additional considerations need to be included.

Most problems with foundations of lightweight structures are caused by the soils below the foundations, rather than high loads from the structure. For example, founda-
8.6 Footings on or Near Slopes

![Diagram of footing on slope]

Vesci's bearing capacity formulas in Chapter 6 are able to consider footings near sloping ground, and we could compute the settlement of such footings. However, it is best to avoid this condition whenever possible. Special concerns for such situations include:

- The reduction in lateral support makes bearing capacity failures more likely.
- The foundation might be undermined if a shallow (or deep!) landslide were to occur.
- The near-surface soils may be slowly creeping downhill, and this creep may cause the footing to move slowly downslope. This is especially likely in clays.

However, there are circumstances where footings must be built on or near a slope. Examples include abutments of bridges supported on approach embankments, foundations for electrical transmission towers, and some buildings.

Shields, Chandler, and Garnier (1990) produced another solution for the bearing capacity of footings located on sandy slopes. This method, based on centrifuge tests, relates the bearing capacity of footings at various locations with that of a comparable footing with $D = 0$ located on level ground. Figures 8.11 to 8.13 give this ratio for 1.5:1 and 2:1 slopes.

The Uniform Building Code and the International Building Code require setbacks as shown in Figure 8.14. We can meet these criteria either by moving the footing away from the slope or by making it deeper.

### QUESTIONS AND PRACTICE PROBLEMS

8.13 A certain square spread footing for an office building is to support the following downward design loads: dead load = 800 kN, live load = 500 kN, seismic load = 400 kN. The 33 percent increase for seismic load capacity is applicable to this site.

a. Compute the design load.

b. Using the design chart from Example 8.2, determine the required width of this footing such that the total settlement is no more than 20 mm.

8.14 A three-story wood-frame building is to be built on a site underlain by sandy clay. This building will have wall loads of 1900 lb/ft on a certain exterior wall. Using the minimum dimensions presented in Table 8.4 and presumptive bearing pressures from the International Building Code as presented in Table 8.5, compute the required width and depth of this footing. Show your final design in a sketch.

### TABLE 8.5 MINIMUM DIMENSIONS FOR CONTINUOUS FOOTINGS THAT SUPPORT WOOD-FRAME BEARING WALLS PER UBC AND IBC (ICBO, 1997 and ICC, 2000)

<table>
<thead>
<tr>
<th>Number of floors supported by the foundation</th>
<th>Thickness of Foundation Wall, $B$ (mm)</th>
<th>Footing Width, $B$ (mm)</th>
<th>Footing Thickness, $T$ (mm)</th>
<th>Footing Depth Below Undisturbed Ground Surface, $D$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>300</td>
<td>12</td>
<td>150</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>375</td>
<td>15</td>
<td>175</td>
</tr>
<tr>
<td>3</td>
<td>250</td>
<td>450</td>
<td>18</td>
<td>200</td>
</tr>
</tbody>
</table>

1. Where unusual conditions or frost conditions are found, footings and foundations shall be as required by UBC Section 1806.1 or IBC Section 1805.2.1.
2. The ground under the floor may be excavated to the elevation of the top of the footing.
3. Interior stud bearing walls may be supported by isolated footings. The footing width and length shall be twice the width shown in this table and the footings shall be spaced not more than 6 ft (1829 mm) on center.
4. In Seismic Zone 4, continuous footings shall be provided with a minimum of one No. 4 bar top and bottom.
5. Foundations may support a roof in addition to the stipulated number of floors. Foundations supporting roofs only shall be as required for supporting one floor.
8.6 Footings on or Near Slopes

Figure 8.11 Definition of terms for computing bearing capacity of footings near or on sandy slopes (Adapted from Shields, Chandler and Garnier, 1990; Used by permission of ASCE).

Figure 8.12 Bearing capacity of footings near or on a 2H:1V sandy slopes. The contours are the bearing capacity divided by the bearing capacity of a comparable footing located at the surface of level ground, expressed as a percentage. (Adapted from Shields, Chandler and Garnier, 1990; Used by permission of ASCE).

Figure 8.13 Bearing capacity of footings near or on a 1.5H:1V sandy slopes. The contours are the bearing capacity divided by the bearing capacity of a comparable footing located at the surface of level ground, expressed as a percentage. (Adapted from Shields, Chandler and Garnier, 1990; Used by permission of ASCE).

Figure 8.14 Footing setback as required by the Uniform Building Code (1806.5) and the International Building Code (1805.3) for slopes steeper than 3 horizontal to 1 vertical. The horizontal distance from the footing to the face of the slope should be at least 0.7H, but need not exceed 40 ft (12 m). For slopes that are steeper than 1 horizontal to 1 vertical, this setback distance should be measured from a line that extends from the toe of the slope at an angle of 45°. (Adapted from the 1997 edition of the Uniform Building Code. © 1997, with the permission of the publisher, the International Conference of Building Officials and the 2000 edition of the International Building Code).
8.7 FOOTINGS ON FROZEN SOILS

In many parts of the world, the air temperature in the winter often falls below the freezing point of water (0°C) and remains there for extended periods. When this happens, the ground becomes frozen. In the summer, the soils become warmer and return to their unfrozen state. Much of the northern United States, Canada, central Europe, and other places with similar climates experience this annual phenomenon.

The greatest depth to which the ground might become frozen at a given locality is known as the depth of frost penetration. This distance is part of an interesting thermodynamics problem and is a function of the air temperature and its variation with time, the initial soil temperature, the thermal properties of the soil, and other factors. The deepest penetrations are obtained when very cold air temperatures are maintained for a long duration. Typical depths of frost penetration in the United States are shown in Figure 8.15.

These annual freeze-thaw cycles create special problems that need to be considered in foundation design.

Frost Heave

The most common foundation problem with frozen soils is frost heave, which is a rising of the ground when it freezes.

When water freezes, it expands about 9 percent in volume. If the soil is saturated and has a typical porosity (say, 40 percent), it will expand about 9% x 40% = 4% in volume when it freezes. In climates comparable to those in the northern United States, this could correspond to surface heaves of as much as 25 to 50 mm (1-2 in). Although such heaves are significant, they are usually fairly uniform, and thus cause relatively little damage.

However, there is a second, more insidious source of frost heave. If the groundwater table is relatively shallow, capillary action can draw water up to the frozen zone and form ice lenses, as shown in Figure 8.16. In some soils, this mechanism can move large quantities of water, so it is not unusual for these lenses to produce ground surface heaves of 300 mm (1 ft) or more. Such heaves are likely to be very irregular and create a hummocky ground surface that could extensively damage structures, pavements, and other civil engineering works.

In the spring, the warmer weather permits the soil to thaw, beginning at the ground surface. As the ice melts, it leaves a soil with much more water than was originally present. Because the lower soils will still be frozen for a time, this water temporarily cannot drain away, and the result is a supersaturated soil that is very weak. This condition is often the cause of ruts and potholes in highways and can also affect the performance of shallow foundations and floor slabs. Once all the soil has thawed, the excess water drains down and the soil regains its strength. This annual cycle is shown in Figure 8.17.

To design foundations in soils that are prone to frost heave, we need to know the depth of frost penetration. This depth could be estimated using Figure 8.15, but as a practical matter it is normally dictated by local building codes. For example, the Chicago Building Code specifies a design frost penetration depth of 1.1 m (42 in). Rarely, if ever, would a rigorous thermodynamic analysis be performed in practice.

Next, the engineer will consider whether ice lenses are likely to form within the frozen zone, thus causing frost heave. This will occur only if both of the following conditions are met:

1. There is a nearby source of water; and
2. The soil is frost-susceptible.

To be considered frost-susceptible, a soil must be capable of drawing significant quantities of water up from the groundwater table into the frozen zone. Clean sands and
gravel is not frost-susceptible because they are not capable of significant capillary rise. Conversely, clays are capable of raising water through capillary rise, but they have a low permeability and are therefore unable to deliver large quantities of water. Therefore, clays are capable of only limited frost heave. However, intermediate soils, such as silts and fine sands, have both characteristics: They are capable of substantial capillary rise and have a high permeability. Large ice lenses are able to form in these soils, so they are considered to be very frost-susceptible.

The U.S. Army Corps of Engineers has classified frost-susceptible soils into four groups, as shown in Table 8.6. Higher group numbers correspond to greater frost susceptibility and more potential for formation of ice lenses. Clean sands and gravels (i.e., <3% finer than 0.02 mm) may be considered non-frost-susceptible and are not included in this table.

The most common method of protecting foundations from the effects of frost heave is to build them at a depth below the depth of frost penetration. This is usually wise in all soils, whether or not they are frost-susceptible and whether or not the groundwater table is nearby. Even "frost-free" clean sands and gravels often have silt lenses that are prone to heave, and groundwater conditions can change unexpectedly, thus introducing new sources of water. The small cost of building deeper foundations is a wise investment in such cases. However, foundations supported on bedrock or interior foundations in heated buildings normally do not need to be extended below the depth of frost penetration.

Builders in Canada and Scandinavia often protect buildings with slab-on-grade floors using thermal insulation, as shown in Figure 8.18. This method traps heat stored in the ground during the summer and thus protects against frost heave, even though the foundations are shallower than the normal frost depth. Both heated and nonheated buildings can use this technique (NAHB, 1988 and 1990).

Alternatively, the natural soils may be excavated to the frost penetration depth and replaced with soils that are known to be frost-free. This may be an attractive alternative for unheated buildings with slab floors to protect both the floor and the foundation from frost heave.

Although frost heave problems are usually due to freezing temperatures from natural causes, it is also possible to freeze the soil artificially. For example, refrigerated buildings such as cold-storage warehouses or indoor ice skating rinks can freeze the soils below and be damaged by frost heave, even in areas where natural frost heave is not a concern (Thorson and Braun, 1975). Placing insulation or air passages between the building and the soil and/or using non-frost-susceptible soils usually prevents these problems.

A peculiar hazard to keep in mind when foundations or walls extend through frost-susceptible soils is adfreezing (CGS, 1992). This is the bonding of soil to a wall or foundation as it freezes. If heaving occurs after the adfreezing, the rising soil will impose a large
Chapter 8  Spread Footings—Geotechnical Design

8.8 Footings on Soils Prone to Scour

Scour is the loss of soil because of erosion in river bottoms or in waterfront areas. This is an important consideration for design of foundations for bridges, piers, docks, and other structures, because the soils around and beneath the foundation could be washed away.

Scour around the foundations is the most common cause of bridge failure. For example, during spring 1987, there were seventeen bridge failures caused by scour in the northeastern United States alone (Huber, 1991). The most notable of these was the collapse of the Interstate Route 90 bridge over Schoharie Creek in New York (Murillo, 1991).

The Alaska Pipeline project is an excellent example of a major engineering work partially supported on permafrost (Luscher et al., 1975).

TABLE 8.6  FROST SUSCEPTIBILITY OF VARIOUS SOILS ACCORDING TO THE U.S. ARMY CORPS OF ENGINEERS (Adapted from Johnston, 1981).

<table>
<thead>
<tr>
<th>Group</th>
<th>Soil Types*</th>
<th>USCS Group Symbolsb</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Gravels with 3–10% finer than 0.02 mm</td>
<td>GW, GP, GW-GM, GP-GM</td>
</tr>
<tr>
<td>(least susceptible)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>a. Gravels with 10–20% finer than 0.02 mm</td>
<td>GM, GM-GM, GP-GM</td>
</tr>
<tr>
<td></td>
<td>b. Sands with 3–15% finer than 0.02 mm</td>
<td>SW, SP, SM, SW-SM, SP-SM</td>
</tr>
<tr>
<td>F3</td>
<td>a. Gravels with more than 20% finer than 0.02 mm</td>
<td>GM, GC</td>
</tr>
<tr>
<td></td>
<td>b. Sands, except very fine silty sands, with more than 15% finer than 0.02 mm</td>
<td>SM, SC</td>
</tr>
<tr>
<td></td>
<td>c. Clays with $I_p &gt; 12$, except varved clays</td>
<td>CL, CH</td>
</tr>
<tr>
<td>F4</td>
<td>a. Silts and sandy silts</td>
<td>ML, MH</td>
</tr>
<tr>
<td>(most susceptible)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b. Fine silty sands with more than 15% finer than 0.02 mm</td>
<td>SM</td>
</tr>
<tr>
<td></td>
<td>c. Lean clays with $I_p &lt; 12$</td>
<td>CL, CL-ML</td>
</tr>
<tr>
<td></td>
<td>d. Varved clays and other fine-grained, banded sediments</td>
<td></td>
</tr>
</tbody>
</table>

* $I_p$ = Plasticity Index (explained in Chapter 3).
*See Chapter 3 for an explanation of USCS group symbols.

Permafrost

In areas where the mean annual temperature is less than 0°C, the penetration of freezing in the winter may exceed the penetration of thawing in the summer and the ground can become frozen to a great depth. This creates a zone of permanently frozen soil known as permafrost. In the harshest of cold climates, such as Greenland, the frozen ground is continuous, whereas in slightly "milder" climates, such as central Alaska, central Canada, and much of Siberia, the permafrost is discontinuous. Areas of seasonal and continuous permafrost in Canada are shown in Figure 8.19.

In areas where the summer thaws occur, the upper soils can be very wet and weak and probably not capable of supporting any significant loads, while the deeper soils remain permanently frozen. Foundations must penetrate through this seasonal zone and well into the permanently frozen ground below. It is very important that these foundations be designed so that they do not transmit heat to the permafrost, so buildings are typically built with raised floors and a ducting system to maintain subfreezing air temperatures between the floor and the ground surface.

The Alaska Pipeline project is an excellent example of a major engineering work partially supported on permafrost (Luscher et al., 1975).

8.8 FOOTINGS ON SOILS PRONE TO SCOUR

Upward load on the structure, possibly separating structural members. Placing a 10-mm (0.5 in) thick sheet of rigid polystyrene between the foundation and the frozen soil reduces the adfreezing potential.

Permafrost

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8.8 FOOTINGS ON SOILS PRONE TO SCOUR

Scour is the loss of soil because of erosion in river bottoms or in waterfront areas. This is an important consideration for design of foundations for bridges, piers, docks, and other structures, because the soils around and beneath the foundation could be washed away.

Scour around the foundations is the most common cause of bridge failure. For example, during spring 1987, there were seventeen bridge failures caused by scour in the northeastern United States alone (Huber, 1991). The most notable of these was the collapse of the Interstate Route 90 bridge over Schoharie Creek in New York (Murillo, 1991).

The Alaska Pipeline project is an excellent example of a major engineering work partially supported on permafrost (Luscher et al., 1975).
Footings on Soils Prone to Scour

8.8

Scour is part of the natural process that moves river-bottom sediments downstream. It can create large changes in the elevation of the river bottom. For example, Murphy (1908) describes a site on the Colorado River near Yuma, Arizona, where the river bed consists of highly erodible fine silty sands and silts. While passing a flood, the water level at this point rose 4.3 m (14 ft) and the bottom soils scoured to depths of up to 11 m (36 ft)! If a bridge foundation located 10.7 m (35 ft) below the river bottom had been built at this location, it would have been completely undermined by the scour and the bridge would have collapsed.

1987), a failure that killed ten people. Figures 8.20 and 8.21 show another bridge that collapsed as a result of scour.

Scour is part of the natural process that moves river-bottom sediments downstream. It can create large changes in the elevation of the river bottom. For example, Murphy (1908) describes a site on the Colorado River near Yuma, Arizona, where the river bed consists of highly erodible fine silty sands and silts. While passing a flood, the water level at this point rose 4.3 m (14 ft) and the bottom soils scoured to depths of up to 11 m (36 ft)! If a bridge foundation located 10.7 m (35 ft) below the river bottom had been built at this location, it would have been completely undermined by the scour and the bridge would have collapsed.

Figure 8.20 One of the mid-channel piers supporting this bridge sank about 1.5 m when it was undermined by scour in the river channel.

Figure 8.21 Deck view of the bridge shown in Figure 8.20. The lanes on the right side of the fence are supported by a separate pier that was not undermined by the scour.
Scour is often greatest at places where the river is narrowest and constrained by levees or other means. Unfortunately, these are the locations most often selected for bridges. The presence of a bridge pier also creates water flow patterns that intensify the scour. However, methods are available to predict scour depths (Richardson et al., 1991) and engineers can use preventive measures, such as armoring, to prevent scour problems (TRB, 1984).

8.9 FOOTINGS ON ROCK

In comparison to foundations on soil, those on bedrock usually present few difficulties for the designer (Peck, 1976). The greatest problems often involve difficulties in construction, such as excavation problems and proper removal of weathered or disturbed material to provide good contact between the footing and the bedrock.

The allowable bearing pressure on rock may be determined in at least four ways (Kulhawy and Goodman, 1980):

- Presumptive allowable bearing pressures
- Empirical rules
- Rational methods based on bearing capacity and settlement analyses
- Full-scale load tests

When supported on good quality rock, spread footings are normally able to support moderately large loads with very little settlement. Engineers usually design them using presumptive bearing pressures, preferably those developed for the local geologic conditions. Typical values are listed in Table 8.7.

If the rock is very strong, the strength of the concrete may govern the bearing capacity of spread footings. Therefore, do not use an allowable bearing value, $q_a$, greater than one-third of the compressive strength of the concrete ($0.33 f_c$).

When working with bedrock, be aware of certain special problems. For example, soluble rocks, including limestone, may have underground cavities that might collapse, causing sinkholes to form at the ground surface. These have caused extensive damage to buildings in Florida and elsewhere.

Soft rocks, such as siltstone, claystone, and mudstone, are very similar to hard soil, and often can be sampled, tested, and evaluated using methods developed for soils.

### TABLE 8.7 TYPICAL PRESumptIVE ALLOWABLE BEARING PRESSURES FOR FOUNDATIONS ON BEDROCK (Adapted from US Navy, 1982b)

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Rock Consistency</th>
<th>Allowable Bearing Pressure, $q_a$ (lb/ft$^2$)</th>
<th>(kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Massive crystalline igneous and metamorphic rock:</td>
<td>Hard and sound (minor cracks OK)</td>
<td>120,000-200,000</td>
<td>6000-10,000</td>
</tr>
<tr>
<td>Granite, diorite, basalt, gneiss, thoroughly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cemented conglomerate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foliated metamorphic rock: Slate, schist</td>
<td>Medium hard, sound (minor cracks OK)</td>
<td>60,000-80,000</td>
<td>3000-4000</td>
</tr>
<tr>
<td>Sedimentary rock: Hard-cemented shales, siltstone,</td>
<td>Medium hard, sound</td>
<td>30,000-50,000</td>
<td>1500-2500</td>
</tr>
<tr>
<td>sandstone, limestone without cavities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weathered or broken bedrock of any kind: compaction</td>
<td>Soft</td>
<td>16,000-24,000</td>
<td>800-1200</td>
</tr>
<tr>
<td>shale or other argillaceous rock in sound condition</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

QUESTIONS AND PRACTICE PROBLEMS

8.15 A 4 ft square, 2 ft deep spread footing carries a compressive column load of 50 k. The edge of this footing is 1 ft behind the top of a 40 ft tall, 2H:1V descending slope. The soil has the following properties: $c = 200$ lb/ft$^2$, $\phi = 31^\circ$, $\gamma = 121$ lb/ft$^3$, and the groundwater table is at a great depth. Compute the factor of safety against a bearing capacity failure and comment on this design.

8.16 Classify the frost susceptibility of the following soils:

a. Sandy gravel (GW) with 3% finer than 0.02 mm.

b. Well graded sand (SW) with 4% finer than 0.02 mm.

c. Silty sand (SM) with 20% finer than 0.02 mm.

d. Fine silty sand (SM) with 35% finer than 0.02 mm.

e. Sandy silt (ML) with 70% finer than 0.02 mm.

f. Clay (CH) with plasticity index $= 60$.

8.17 A compacted fill is to be placed at a site in North Dakota. The following soils are available for import: Soil 1 - silty sand; Soil 2 - lean clay; Soil 3 - Gravelly coarse sand. Which of these soils would be least likely to have frost heave problems?

8.18 Would it be wise to use slab-on-grade floors for houses built on permafrost? Explain.

8.19 What is the most common cause of failure in bridges?

8.20 A single-story building is to be built on a sandy silt in Detroit. How deep must the exterior footings be below the ground surface to avoid problems with frost heave?
Chapter 8 Spread Footings—Geotechnical Design

8.9 Footings on Rock

Summary

1. The depth of embedment, $D$, must be great enough to accommodate the required footing thickness, $T$. In addition, certain geotechnical concerns, such as loose soils or frost heave, may dictate an even greater depth.

2. The required footing width, $B$, is a geotechnical problem, and is governed by bearing capacity and settlement criteria. It is inconvenient to satisfy these criteria by performing custom bearing capacity and settlement computations for each footing, so we present the results of generic computations in a way that is applicable to the entire site. There are two methods of doing so: the allowable bearing pressure method and the design chart method.

3. Footings subjected to eccentric or moment loads need to be evaluated using the "equivalent footing." 

4. Footings can resist applied shear loads through passive pressure and sliding friction.

5. Wind and seismic loads are normally treated as equivalent static loads. For most soils, load combinations that include wind or seismic components may be evaluated using a 33 percent greater allowable bearing pressure.

6. The design of lightly-loaded footings is often governed by minimum practical dimensions.

7. Lightly-loaded footings are often designed using an presumptive allowable bearing pressure, which is typically obtained from the applicable building code.

8. The design of footings on or near slopes needs to consider the sloping ground.

9. Footings on frozen soils need special considerations. The most common problem is frost heave, and the normal solution is to place the footing below the depth of frost penetration.

10. Footings in or near riverbeds are often prone to scour, and must be designed accordingly.

11. Rock usually provides excellent support for spread footings. Such footings are typically designed using a presumptive allowable bearing pressure.

Vocabulary

<table>
<thead>
<tr>
<th>Allowable bearing pressure</th>
<th>Equivalent footing</th>
<th>Permafrost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentric downward load</td>
<td>Frost-susceptible soil</td>
<td>Presumptive allowable bearing pressure</td>
</tr>
<tr>
<td>Depth of frost penetration</td>
<td>Lightly-loaded footing</td>
<td>Scour</td>
</tr>
<tr>
<td>Design chart</td>
<td>Moment load</td>
<td>Shear load</td>
</tr>
<tr>
<td>Eccentric load</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8.22 The soil at a certain site has the following geotechnical design parameters: $q_c = 4000$ lb/ft$^2$, $\mu_s = 0.28$, and $\lambda_s = 200$ lb/ft$^2$. The groundwater table is at a depth of 20 ft. A column that is to

Figure 8.22 CPT data and synthesis of boring for Problems 8.23 and 8.24.
be supported on a square spread footing on this soil will impart the following load combinations onto the footing: $P = 200\, k$, $V = 18\, k$.

Determine the required footing width and depth of embedment.

8.23 Six cone penetration tests and four exploratory borings have been performed at the site of a proposed warehouse building. The underlying soils are natural sands and silty sands with occasional gravel. The CPT results and a synthesis of the borings are shown in Figure 8.22. The warehouse will be supported on 3 ft deep square footings that will have design downward loads of 100 to 600 k. The allowable total settlement is 1.0 in and the allowable differential settlement is 0.5 in. Using this data with reasonable factors of safety, develop values of $q_{A}$, $p_{A}$, and $\lambda_{V}$. Use Figure 4.16 to estimate the friction angle.

8.24 Using the design values in Problem 8.23, determine the required width of a footing that must support the following load combinations:

- **Load combination 1**: $P_D = 200\, k$, $P_L = 0$, $V = 0$
- **Load combination 2**: $P_D = 200\, k$, $P_L = 0$, $V = 21\, k$
- **Load combination 3**: $P_D = 200\, k$, $P_L = 240$, $V = 40\, k$
- **Load combination 4**: $P_D = 200\, k$, $P_L = 240$, $P_E = -20\, k$, $V_E = 40\, k$
The plan dimensions and minimum embedment depth of spread footings are primarily geotechnical concerns, as discussed in Chapters 6 to 8. Once these dimensions have been set, the next step is to develop a structural design that gives the foundation enough integrity to safely transmit the design loads from the structure to the ground. The structural design process for reinforced concrete foundations includes:

- Selecting a concrete with an appropriate strength
- Selecting an appropriate grade of reinforcing steel
- Determining the required foundation thickness, \( T \), as shown in Figure 9.1
- Determining the size, number, and spacing of the reinforcing bars
- Designing the connection between the superstructure and the foundation.

The structural design aspects of foundation engineering are far more codified than are the geotechnical aspects. These codes are based on the results of research, the performance of existing structures, and the professional judgment of experts. Engineers in North America use the Building Code Requirements for Structural Concrete (ACI 318-99 and ACI 318M-99) for most projects. This code is published by the American Concrete Institute (ACI, 1999). The most notable alternative to ACI is the Standard Specifications for Highway Bridges published by the American Association of State Highway and Transportation Officials (AASHTO, 1996). The model building codes (ICBO, 1997; BOCA, 1996; SBCCI, 1997; ICC, 2000) contain additional design requirements.

This chapter covers the major principles of structural design of spread footings, and often refers to specific code requirements, with references shown in brackets [ ]. However, it is not a comprehensive discussion of every code provision, and thus is not a substitute for the code books.

### 9.1 SELECTION OF MATERIALS

Unlike geotechnical engineers, who usually have little or no control over the engineering properties of the soil, structural engineers can, within limits, select the engineering properties of the structural materials. In the context of spread footing design, we must select an appropriate concrete strength, \( f'_c \), and reinforcing steel strength, \( f_y \).

When designing a concrete superstructure, engineers typically use concrete that has \( f'_c = 20-35 \text{ MPa} (3000-5000 \text{ lb/in}^2) \). In very tall structures, \( f'_c \) might be as large as 70 \text{ MPa} (10,000 \text{ lb/in}^2). The primary motive for using high-strength concrete in the superstructure is that it reduces the section sizes, which allows more space for occupancy and reduces the weight of the structure. These reduced member weights also reduce the dead loads on the underlying members.

However, the plan dimensions of spread footings are governed by bearing capacity and settlement concerns and will not be affected by changes in the strength of the concrete; only the thickness, \( T \), will change. Even then, the required excavation depth, \( D \), may or may not change because it might be governed by other factors. In addition, saving weight in a footing is of little concern because it is the lowest structural member and does not affect the dead load on any other members. In fact, additional weight may actually be a benefit in that it increases the uplift capacity.

Because of these considerations, and because of the additional materials and inspection costs of high strength concrete, spread footings are usually designed using an \( f'_c \) of only 15–20 \text{ MPa} (2000–3000 \text{ lb/in}^2). For footings that carry relatively large loads, perhaps greater than about 2000 \text{kN} (500 \text{k}), higher strength concrete might be justified to keep the footing thickness within reasonable limits, perhaps using an \( f'_c \) as high as 35 \text{ MPa} (5000 \text{ lb/in}^2).

Since the flexural stresses in footings are small, grade 40 steel (metric grade 300) is usually adequate. However, this grade is readily available only in sizes up through #6 (metric #22), and grade 60 steel (metric grade 420) may be required on the remainder of the project. Therefore, engineers often use grade 60 (metric grade 420) steel in the footings for reinforced concrete buildings so only one grade of steel is used on the project. This makes it less likely that leftover grade 40 (metric grade 300) bars would accidentally be placed in the superstructure.

### 9.2 BASIS FOR DESIGN METHODS

Before the twentieth century, the design of spread footings was based primarily on precedent. Engineers knew very little about how footings behaved, so they followed designs that had worked in the past.
Chapter 9 Spread Footings—Structural Design

The first major advance in our understanding of the structural behavior of reinforced concrete footings came as a result of full-scale load tests conducted at the University of Illinois by Talbot (1913). He tested 197 footings in the laboratory and studied the mechanisms of failure. These tests highlighted the importance of shear in footings.

During the next five decades, other individuals in the United States, Germany, and elsewhere conducted additional tests. These tests produced important experimental information on the flexural and shear resistance of spread footings and slabs as well as the response of new and improved materials. Richart's (1948) tests were among the most significant of these. He tested 156 footings of various shapes and construction details by placing them on a bed of automotive coil springs that simulated the support from the soil and loaded them using a large testing machine until they failed. Whitney (1957) and Moe (1961) also made important contributions.

A committee of engineers (ACI-ASCE, 1962) synthesized this data and developed the analysis and design methodology that engineers now use. Because of the experimental nature of its development, this method uses simplified, and sometimes arbitrary, models of the behavior of footings. It also is conservative.

As often happens, theoretical studies have come after the experimental studies and after the establishment of design procedures (Jiang, 1983; Rao and Singh, 1987). Although work of this type has had some impact on engineering practice, it is not likely that the basic approach will change soon. Engineers are satisfied with the current procedures for the following reasons:

- Spread footings are inexpensive, and the additional costs of a conservative design are small.
- The additional weight that results from a conservative design does not increase the dead load on any other member.
- The construction tolerances for spread footings are wider than those for the superstructure, so additional precision in the design probably would be lost during construction.
- Although perhaps crude when compared to some methods available to analyze superstructures, the current methods are probably more precise than the geotechnical analyses of spread footings and therefore are not the weak link in the design process.
- Spread footings have performed well from a structural point-of-view. Failures and other difficulties have usually been due to geotechnical or construction problems, not bad structural design.
- The additional weight of conservatively designed spread footings provides more resistance to uplift loads.

Standard design methods emphasize two modes of failure: shear and flexure. A shear failure, shown in Figure 9.2, occurs when part of the footing comes out of the bottom. This type of failure is actually a combination of tension and shear on inclined failure surfaces. We resist this mode of failure by providing an adequate footing thickness, $T$. A flexural failure is shown in Figure 9.3. We analyze this mode of failure by treating the footing as an inverted cantilever beam and resisting the flexural stresses by placing tensile steel reinforcement near the bottom of the footing.

9.3 DESIGN LOADS

The structural design of spread footings is based on LRFD methods (ACI calls it ultimate strength design or USD), and thus uses the factored loads as defined in Equations 2.7 to 2.15. Virtually all footings support a compressive load, $P$, and it should be computed...
without including the weight of the footing because this weight is evenly distributed and thus does not produce shear or moment in the footing. Some footings also support shear ($V_u$) and/or moment ($M_u$) loads, as shown in Figure 9.1, both of which must be expressed as the factored load. This is often a point of confusion, because the geotechnical design of the same footing is normally based on ASD methods, and thus use the unfactored load, as defined in Equations 2.1 to 2.4. In addition, the geotechnical design must include the weight of the footing.

Therefore, when designing spread footings, be especially careful when computing the load. The footing width, $B$, is based on geotechnical requirements and is thus based on the unfactored load, as discussed in Chapter 8, whereas the thickness, $T$, and the reinforcement are structural concerns, and thus are based on the factored load. Examples 9.1 and 9.2 illustrate the application of these principles.

### 9.4 MINIMUM COVER REQUIREMENTS AND STANDARD DIMENSIONS

The ACI code specifies the minimum amount of concrete cover that must be present around all steel reinforcing bars [7.7]. For concrete in contact with the ground, such as spread footings, at least 70 mm (3 in) of concrete cover is required, as shown in Figure 9.4. This cover distance is measured from the edge of the bars, not the centerlines. It provides proper anchorage of the bars and corrosion protection. It also allows for irregularities in the excavation and accommodates possible contamination of the lower portion of the concrete.

Sometimes it is appropriate to specify additional cover between the rebar and the soil. For example, it is very difficult to maintain smooth footing excavation at sites with loose sands or soft clays, so more cover may be appropriate. Sometimes contractors place a thin layer of lean concrete, called a mud slab or a leveling slab, in the bottom of the footing excavation at such sites before placing the steel, thus providing a smooth working surface.

For design purposes, we ignore any strength in the concrete below the reinforcing steel. Only the concrete between the top of the footing and the rebars is considered in our analyses. This depth is the effective depth, $d$, as shown in Figure 9.4.

### 9.5 SQUARE FOOTINGS

Footings are typically excavated using backhoes, and thus do not have precise as-built dimensions. Therefore, there is no need to be overly precise when specifying the footing thickness $T$. Round it to a multiple of 3 in or 100 mm (i.e., 12, 15, 18, 21, etc. inches or 300, 400, 500, etc. mm). The corresponding values of $d$ are:

$$d = T - 3\text{ in} - d_e$$  \hspace{1cm} (9.1 \text{ English})

$$d = T - 70\text{ mm} - d_e$$  \hspace{1cm} (9.1 \text{ SI})

Where $d_e$ is the nominal diameter of the steel reinforcing bars (see Table 9.1).

ACI [15.7] requires $d$ be at least 6 in (150 mm), so the minimum acceptable $T$ for reinforced footings is 12 in or 300 mm.

### TABLE 9.1 DESIGN DATA FOR STEEL REINFORCING BARS

<table>
<thead>
<tr>
<th>Bar Size Designation</th>
<th>Available Grades</th>
<th>Nominal Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>English</td>
<td>SI</td>
</tr>
<tr>
<td>#3</td>
<td>#10</td>
<td>40</td>
</tr>
<tr>
<td>#4</td>
<td>#13</td>
<td>40</td>
</tr>
<tr>
<td>#5</td>
<td>#16</td>
<td>40</td>
</tr>
<tr>
<td>#6</td>
<td>#19</td>
<td>40</td>
</tr>
<tr>
<td>#7</td>
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<td>60</td>
</tr>
<tr>
<td>#8</td>
<td>#25</td>
<td>60</td>
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<td>#9</td>
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</tr>
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<tr>
<td>#11</td>
<td>#36</td>
<td>60</td>
</tr>
<tr>
<td>#14</td>
<td>#43</td>
<td>60</td>
</tr>
<tr>
<td>#18</td>
<td>#57</td>
<td>60</td>
</tr>
</tbody>
</table>

Flexural Steel 70 mm or 3 in

Figure 9.4 In square spread footings, the effective depth is the distance from the top of the concrete to the contact point of the flexural steel.
spread footings, and because we neglect the shear strength of the flexural steel. The only source of shear resistance is the concrete above the flexural reinforcement, so the effective depth, \( d \), as shown in Figure 9.4, must be large enough to provide sufficient shear capacity. We then perform the flexural analysis using this value of \( d \).

**Designing for Shear**

ACI defines two modes of shear failure, one-way shear (also known as beam shear or wide-beam shear) and two-way shear (also known as diagonal tension shear). In the context of spread footings, these two modes correspond to the failures shown in Figure 9.5. Although the failure surfaces are actually inclined, as shown in Figure 9.2, engineers use these idealized vertical surfaces to simplify the computations.

**Figure 9.5** The two modes of shear failure: (a) one-way shear, and (b) two-way shear.

Various investigators have suggested different locations for the idealized critical shear surfaces shown in Figure 9.5. The ACI code [11.12.1] specifies that they be located a distance \( d \) from the face of the column for one-way shear and a distance \( d/2 \) for two-way shear.

The footing design is satisfactory for shear when it satisfies the following condition on all critical shear surfaces:

\[
V_{\text{u}} \leq \phi V_{\text{c}}
\]  

Where:
- \( V_{\text{u}} \) = factored shear force on critical surface
- \( \phi \) = resistance factor for shear = 0.85
- \( V_{\text{c}} \) = nominal shear load capacity on the critical surface

The nominal shear load capacity, \( V_{\text{c}} \), on the critical shear surface is [11.1]:

\[
V_{\text{c}} = V_{\text{c}} + V_{\text{s}}
\]  

Where:
- \( V_{\text{c}} \) = nominal shear load capacity of concrete
- \( V_{\text{s}} \) = nominal shear load capacity of reinforcing steel

For spread footings, we neglect \( V_{\text{s}} \) and rely only on the concrete for shear resistance.

**Two-Way Shear**

The footing may be subjected to applied normal, moment, and shear loads, \( P_{\text{u}}, M_{\text{u}}, \) and \( V_{\text{u}} \), all of which produce shear forces on the critical shear surfaces.

To visualize the shear force on the critical surface, \( V_{\text{u}} \), caused by the applied normal load, \( P_{\text{u}} \), we divide the footing into two blocks, one inside the shear surface and the other outside, as shown in Figure 9.6. The factored normal load, \( P_{\text{u}} \), is applied to the top of the inner block and is transferred to a uniform pressure acting on the base of both blocks. Some of this load is transferred to the soil beneath the inner block, while the remainder must pass through the critical shear surface and enters the soil beneath the lower block. Only the later portion produces a shear force on the critical shear surface. In other words, the percentage of \( P_{\text{u}} \) that produces shear along the critical surfaces is the ratio of the base area of the outer block to the total base area.

If an applied moment load, \( M_{\text{u}} \), is present, it produces an additional shear force on two opposing faces of the inner block, as shown in Figure 9.7. The shear force on one of the faces acts in the same direction as the shear force induced by the normal load, while that on the other face acts in the opposite direction. Therefore, the face with both forces
Figure 9.6 The inner block is the portion of the footing inside the critical section for two-way shear. The factored shear force acting along the perimeter of this block, $V_{cu}$, must not exceed $\phi V_u$. The factored shear force, $V_{cu}$, is the portion of the factored column load, $P_u$, that must pass through the outside surfaces of the inner block before reaching the ground.

The design should be based on the larger of the $V_{cu}$ values obtained from Equations 9.4 and 9.5, thus accounting for applied normal, shear, and/or moment loads.

If an applied shear load, $V_u$, is present and it acts in the same direction as the moment load (which is the usual case), it produces a shear force on the other two faces, as shown in Figure 9.8. If we assume the applied shear force is evenly divided between these two faces, then the shear force on each face is:

$$V_{wc} = \left( \frac{P_u}{4} + \frac{M_u}{c + d} \right) \left( \frac{\text{base area of outer block}}{\text{total base area}} \right)$$

(9.4)

$$= \left( \frac{P_u}{4} + \frac{M_u}{c + d} \right) \left( \frac{B^2 - (c + d)^2}{B^2} \right)$$

$V_u$ acting in the same direction has the greatest shear force, and thus controls the design. The force on this face is:

$$V_{wc} = \left( \frac{P_u}{4} + \frac{M_u}{c + d} \right) \left( \frac{\text{base area of outer block}}{\text{total base area}} \right)$$

(9.4)

$$= \left( \frac{P_u}{4} + \frac{M_u}{c + d} \right) \left( B^2 - (c + d)^2 \right)$$

(9.5)

Where:

- $V_{wc}$ = factored shear force on the most critical face
- $P_u$ = applied normal load
- $M_u$ = applied moment load
- $V_u$ = applied shear load
- $c$ = column width or diameter (for concrete columns) or base plate width (for steel columns)
- $d$ = effective depth
- $B$ = footing width

The design should be based on the larger of the $V_{wc}$ values obtained from Equations 9.4 and 9.5, thus accounting for applied normal, shear, and/or moment loads.
Chapter 9 Spread Footings—Structural Design

For square footings supporting square or circular columns located in the interior of the footing (i.e., not on the edge or corner), the nominal two-way shear capacity is [ACI 11.12.2.1]:

\[ V_{nc} = V_c = 4 b_o d \sqrt{f'_c} \]  
(9.6 English)

\[ V_{nc} = V_c = \frac{1}{3} b_o d \sqrt{f'_c} \]  
(9.6 SI)

Where:
- \( V_{nc} \) = nominal two-way shear capacity on the critical section (lb, N)
- \( V_c \) = nominal two-way shear capacity of concrete (lb, N)
- \( b_o \) = length of critical shear surface = length of one face of inner block (in, mm)
- \( d \) = effective depth (in, mm)
- \( f'_c \) = 28-day compressive strength of concrete (lb/in², MPa)

Other criteria apply if the column has another shape, or if it is located along edge or corner of the footing [ACI 11.12.2.1]. Special criteria also apply if the footing is made of prestressed concrete [ACI 11.12.2.2], but spread footings are rarely, if ever, prestressed.

The objective of this analysis is to find the effective depth, \( d \), that satisfies Equation 9.2. Both \( V_{nc} \) and \( V_c \) depend on the effective depth, \( d \), but there is no direct solution. Therefore, it is necessary to use the following procedure:

1. Assume a trial value for \( d \). Usually a value approximately equal to the column width is a good first trial. When selecting trial values of \( d \), remember \( d \) must be a multiple of 3 in or 100 mm, as discussed in Section 9.4, so the corresponding values of \( d \) are the only ones worth considering. Assuming \( d_o = 1 \) in (25 mm), the potential values of \( d \) are 8, 11, 14, 17, etc. inches or 200, 300, 400, etc. mm.
2. Compute \( V_{nc} \) and \( V_c \), and check if Equation 9.2 has been satisfied.
3. Repeat Steps 1 and 2 as necessary until finding the smallest \( d \) that satisfies Equation 9.2.
4. Using Equation 9.1 with \( d_o = 1 \) in or 25 mm, compute the footing thickness, \( T \). Express it as a multiple of 3 in or 100 mm. \( T \) must be at least 12 in or 300 mm.

The final value of \( d_o \) will be determined as a part of the flexural analysis, and may be different from the 1 in or 25 mm assumed here. However, this difference is small compared to the construction tolerances, so there is no need to repeat the shear analysis with the revised \( d_o \).

9.5 Square Footings

One-Way Shear

Two-way shear always governs the design of square footings subjected only to vertical loads. There is no need to check one-way shear in such footings. However, if applied shear and/or moment loads are present, both kinds of shear need to be checked.

To analyze this situation, we will make the following assumptions:

- The shear stress caused by the applied vertical load, \( P_n \), is uniformly distributed across the two critical vertical planes shown in Figure 9.5a.
- The shear stress on the vertical planes caused by the applied moment load, \( M_n \), is expressed by the flexure formula, \( \sigma = M_c / I \), and thus is greatest at the left and right edges of these planes.
- The shear stress caused by the applied shear load is uniformly distributed across the planes.
- The applied normal, moment, and shear loads must be multiplied by \( (B - c - 2d) / B \) before applying them to the critical vertical planes. This factor is the ratio of the footing base area outside the critical planes to the total area, and thus reflects the percentage of the applied loads that must be transmitted through the critical vertical planes.
- The maximum shear stress on the critical vertical surfaces is the vector sum of those due to the applied normal, moment, and shear loads.
- The factored shear stress on the critical vertical surfaces is the greatest shear stress multiplied by the area of the shear surfaces. This may be greater than the integral of the shear stress across the shear surfaces, but is useful because it produces a design that keeps the maximum shear stress within acceptable limits.

Based on these assumptions, we compute the factored shear force on the critical vertical surfaces, \( V_{nc} \), as follows:

\[ V_{nc} = \left( \frac{B - c - 2d}{B} \right) \sqrt{\left( \frac{P_n + 6 M_n}{B} \right)^2 + V_s^2} \]  
(9.8)

Where:
- \( V_{nc} \) = shear force on critical shear surfaces
- \( B \) = footing width
- \( c \) = column width
- \( d \) = effective depth
- \( P_n \) = applied normal load
- \( M_n \) = applied moment load
- \( V_s \) = applied shear load
The nominal one-way shear load capacity on the critical section \([11.3.1.1]\) is:

\[
\begin{align*}
V_{nc} &= V_c = 2 b_c d \sqrt{f_c'} \\
V_{wc} &= V_c = \frac{1}{6} b_c d \sqrt{f_c'}
\end{align*}
\]  

(9.9 English)  

(9.9 SI)

Where:

- \(V_{nc}\) = nominal one-way shear capacity on the critical section (lb, N)
- \(V_c\) = nominal one-way shear capacity of concrete (lb, N)
- \(b_c\) = length of critical shear surface = 2\(B\) (in, mm)
- \(d\) = effective depth (in, mm)
- \(f_c'\) = 28-day compressive strength of concrete (lb/in², MPa)

Once again, the design is satisfactory when Equation 9.2 has been satisfied.

Both \(V_{nc}\) and \(V_{wc}\) depend on the effective depth, \(d\), which must be determined using Equations 9.8 and 9.9 with the procedure described under two-way shear. The final design value of \(d\) is the larger of that obtained from the one-way and two-way shear analyses.

The final value of \(d\) will be determined as a part of the flexural analysis, and may be different from the 1 in or 25 mm assumed here. However, this difference is small compared to the construction tolerances, so there is no need to repeat the shear analysis with the revised \(d\).

Example 9.1—Part A

A 21-inch square reinforced concrete column carries a vertical dead load of 380 k and a vertical live load of 270 k. It is to be supported on a square spread footing that will be founded on a soil with an allowable bearing pressure of 6500 lb/ft². The groundwater table is well below the bottom of the footing. Determine the required width, \(B\), thickness, \(T\), and effective depth, \(d\).

Solution

Unfactored load—Equation 2.2 governs

\[ P = P_d + P_L + \ldots = 380 \text{ k} + 270 \text{ k} + 0 = 650 \text{ k} \]

Per Table 8.1, use \(D = 36\) in

\[ W_f = B^2 D_f = B^2 (3 \text{ ft})(150 \text{ lb/ft}^2) = 450 \text{ lb}^2 \]

\[ B = \sqrt{\frac{P + W_f}{q_s + u_o}} = \sqrt{\frac{650,000 + 450 \text{ lb}^2}{6500 + 0}} \]

\[ B = 10.36 \text{ ft} \rightarrow \text{use } B = 10 \text{ ft 6 in} \ (126 \text{ in}) \]

Factored load (Equation 2.7 governs)

\[ P_s = 1.4 P_d + 1.7 P_L = (1.4)(380) + (1.7)(270) = 991 \text{ k} \]

Because of the large applied load and because this is a large spread footing, we will use \(f'_c = 4000 \text{ lb/in}^2\) and \(f_s' = 60 \text{ k/lin}^2\).

Since there are no applied moment or shear loads, there is no need to check one-way shear. Determine required thickness based on a two-way shear analysis.

Try \(T = 24\) in:

\[ d = T - 1 \text{ bar diameter} - 3 \text{ in} = 24 - 1 - 3 = 20 \text{ in} \]

\[ V_{nc} = \frac{P_s + M_e}{4} \left( \frac{B^2}{c + d} \right) \left( \frac{B^2}{B^2} \right) \]

\[ = \left( \frac{991,000 \text{ lb}}{4} + 0 \right) \left( \frac{(126 \text{ in})^2 - (21 \text{ in} + 20 \text{ in})^2}{(126 \text{ in})^2} \right) \]

\[ = 221,500 \text{ lb} \]

\[ b_0 = c + d = 21 + 20 = 41 \text{ in} \]

\[ V_{wc} = 4 b_0 d \sqrt{f_c'} \]

\[ = 4(41 \text{ in})(20 \text{ in}) \sqrt{4000 \text{ lb/in}^2} \]

\[ = 207,400 \text{ lb} \]

\[ \Phi V_{nc} = (0.85)(207,400 \text{ lb}) = 176,300 \text{ lb} \]

\[ V_{wc} > \Phi V_{nc} \therefore \text{Not acceptable} \]

Try \(T = 27\) in:

\[ d = T - 1 \text{ bar diameter} - 3 \text{ in} = 27 - 1 - 3 = 23 \text{ in} \]

\[ V_{nc} = \frac{P_s + M_e}{4} \left( \frac{B^2}{c + d} \right) \left( \frac{B^2}{B^2} \right) \]

\[ = \left( \frac{991,000 \text{ lb}}{4} + 0 \right) \left( \frac{(126 \text{ in})^2 - (21 \text{ in} + 23 \text{ in})^2}{(126 \text{ in})^2} \right) \]

\[ = 217,500 \text{ lb} \]

\[ b_0 = c + d = 21 + 23 = 44 \text{ in} \]
Chapter 9 Spread Footings—Structural Design

\[ V_{uc} = 4 b_d d \sqrt{f'_c} \]
\[ = 4(41 \text{ in})(23 \text{ in}) \sqrt{4000 \text{ lb/in}^2} \]
\[ = 256,000 \text{ lb} \]
\[ \phi V_{uc} = (0.85)(256,000 \text{ lb}) = 217,600 \text{ lb} \]

\[ V_{uc} = \phi V_{uc} \text{ OK} \]

Note 1: In this case, \( V_{uc} \) is almost exactly equal to \( \phi V_{uc} \). However, since we are considering only certain values of \( d \), it is unusual for them to match so closely.

Note 2: The depth of embedment of 3 ft, as obtained from Table 8.1, is not needed here (unless frost depth or other concerns dictate it). We could use \( D = 30 \) in and still have plenty of room for a 27 inch-thick footing.

Designing for Flexure

Once we have completed the shear analysis, the design process can move to the flexural analysis.

ACI Flexural Design Standards

Reinforcing Steel

Concrete is strong in compression, but weak in tension. Therefore, engineers add reinforcing steel, which is strong in tension, to form reinforced concrete. This reinforcement is necessary in members subjected to pure tension, and those that must resist flexure (bending). Reinforcing steel may consist of either deformed bars (more commonly known as reinforcing bars, or rebars) or welded wire fabric. However, wire fabric is rarely used in foundations.

Manufacturers produce reinforcing bars in various standard diameters, typically ranging between 9.5 mm (3/8 in) and 57.3 mm (2 1/4 in). In the United States, the English and metric bars are the same size (i.e., we have used a soft conversion), and are identified by the bar size designations in Table 9.1.

Rebars are available in various strengths, depending on the steel alloys used to manufacture them. The two most common bar strengths used in the United States are:

- Grade 40 bars (also known as metric grade 300), which have a yield strength, \( f_y \), of 40 kN/m² (300 MPa)
- Grade 60 bars (also known as metric grade 420), which have a yield strength, \( f_y \), of 60 kN/m² (420 MPa)

9.5 Square Footings

Flexural Design Principles

The primary design problem for flexural members is as follows: Given a factored moment on the critical surface, \( M_{uc} \), determine the necessary dimensions of the member and the necessary size and location of reinforcing bars. Fortunately, flexural design in foundations is simpler than that for some other structural members because geotechnical concerns dictate some of the dimensions.

The amount of steel required to resist flexure depends on the effective depth, \( d \), which is the distance from the extreme compression fiber to the centroid of the tension reinforcement, as shown in Figure 9.9.

The nominal moment capacity of a flexural member made of reinforced concrete with \( f'_c \leq 30 \text{ MPa} (4000 \text{ lb/in}^2) \) as shown in Figure 9.9 is:

\[ M_e = A_s \left( \frac{d - a}{2} \right) \quad (9.10) \]
\[ a = \frac{0.85 f'_c d}{f_y} \quad (9.11) \]
\[ \rho = \frac{A_s}{b d} \quad (9.12) \]

Setting \( M_e = \phi M_{uc} \), where \( M_{uc} \) is the factored moment at the section being analyzed, and solving for \( A_s \) gives:

\[ A_s = \left( \frac{f'_c b}{1.176 f_y} \right) \left( d - \sqrt{d^2 - \frac{2.353 M_{uc}}{0.85 f'_c b}} \right) \quad (9.13) \]
Where:

- $A_r =$ cross-sectional area of reinforcing steel
- $f_{cr}' =$ 28-day compressive strength of concrete
- $f_y =$ yield strength of reinforcing steel
- $\rho =$ steel ratio
- $b =$ width of flexural member
- $d =$ effective depth
- $\phi = 0.9$ for flexure in reinforced concrete
- $M_{u} =$ factored moment at the section being analyzed

Two additional considerations also enter the design process: minimum steel and maximum steel. The minimum steel in footings is governed by ACI 10.5.4 and 7.12.2, because footings are treated as “structural slabs of uniform thickness” (MacGregor, 1996). These requirements are as follows:

For grade 40 (metric grade 300) steel

$$A_r \geq 0.0020 \ A_y$$

For grade 60 (metric grade 420) steel

$$A_r \geq 0.0018 \ A_y$$

Development Length

The rebars must extend a sufficient distance into the concrete to develop proper anchorage [ACI 15.6]. This distance is called the development length. Provides the clear spacing between the bars must be at least equal to $d_{bn}$, 25 mm (1 in), or 4/3 times the nominal maximum aggregate size [3.3.2 and 7.6.1], whichever is greater.

The center-to-center spacing of the reinforcement must not exceed 37 or 500 mm (18 in), whichever is less [10.5.4].

Notice how one of these criteria is based on the “clear space” which is the distance between the edges of two adjacent bars, while the other is based on the center-to-center spacing, which is the distance between their centerlines.

For spread footings, use $K_{tr} = 0$, which is conservative.

Where:

- $l_d =$ minimum required development length (in, mm)
- $d_{bn} =$ nominal bar diameter (in, mm)
- $f_y =$ yield strength of reinforcing steel (lb/in$^2$, MPa)
- $f_{cr}' =$ yield strength of transverse reinforcing steel (lb/in$^2$, MPa)
- $f_{cr}' =$ 28-day compressive strength of concrete (lb/in$^2$, MPa)
- $\alpha =$ reinforcement location factor
- $\beta =$ coating factor
- $\gamma =$ reinforcement factor
- $\lambda =$ lightweight concrete factor
- $c =$ spacing or cover dimension (in, mm) = the smaller of the distance from the center of the bar to the nearest concrete surface or one-half the center-to-center spacing of the bar
- $A_r =$ total cross-sectional area of all transverse reinforcement that is within the spacing $s$ and which crosses the potential plane of splitting through the re-

\[
\frac{l_d}{d_{bn}} = 3 \frac{f_y}{40 \sqrt{f_{cr}' \left(\frac{c + K_{tr}}{d_{bn}}\right)}}
\]  

(9.14 English)

\[
\frac{l_d}{d_{bn}} = 9 \frac{f_y}{10 \sqrt{f_{cr}' \left(\frac{c + K_{tr}}{d_{bn}}\right)}}
\]  

(9.14 SI)

\[
K_{tr} = \frac{A_{nt} f_y}{1500 \ s \ n}
\]  

(9.15 English)

\[
K_{tr} = \frac{A_{nt} f_y}{10 \ s \ n}
\]  

(9.15 SI)
inforcement being developed (in\(^2\), mm\(^2\))—may conservatively be taken to be zero.

\[ s = \text{maximum center-to-center spacing of transverse reinforcement within } l_d \]

(in, mm)

The term \((c + K_v)ve\) must be no greater than 2.5, and the product of \(s\) need not exceed 1.7.

In addition, the development length must always be at least 300 mm (12 in).

**Application to Spread Footings**

**Principles**

A square footing bends in two perpendicular directions as shown in Figure 9.10a, and therefore might be designed as a **two-way slab** using methods similar to those that might be applied to a floor slab that is supported on all four sides. However, for practical purposes, it is customary to design footings as if they were a **one-way slab** as shown in Figure 9.10b. This conservative simplification is justified because of the following:

- The full-scale load tests on which this analysis method is based were interpreted this way.
- It is appropriate to design foundations more conservatively than the superstructure.
- The flexural stresses are low, so the amount of steel required is nominal and often governed by \(\rho_{\text{min}}\).
- The additional construction cost due to this simplified approach is nominal.

Once we know the amount of steel needed to carry the applied load in one-way bending, we place the same steel area in the perpendicular direction. In essence the footing is reinforced twice, which provides more reinforcement than required by a more rigorous two-way analysis.

**Steel Area**

The usual procedure for designing flexural members is to prepare a moment diagram and select an appropriate amount of steel for each portion of the member. However, for simple spread footings, we again simplify the problem and design all the steel for the moment that occurs at the **critical section for bending**. The location of this section for various types of columns is shown in Figure 9.11.

We can simplify the computations by defining a distance \(l\), measured from the critical section to the outside edge of the footing. In other words, \(l\) is the cantilever distance. It is computed using the formulas in Table 9.2.

The factored bending moment at the critical section, \(M_{uc}\), is:

\[
M_{uc} = \frac{P_x l^2}{2B} + \frac{2M_i}{B} \tag{9.16}
\]

**Figure 9.10** (a) A spread footing is actually a two-way slab, bending in both the "north-south" and "east-west" directions; (b) For purposes of analysis, engineers assume that the footing is a one-way slab that bends in one axis only.

Where:

- \(M_{uc}\) = factored moment at critical section for bending
- \(P_x\) = factored compressive load from column
- \(M_i\) = factored moment load from column
- \(l\) = cantilever distance (from Table 9.2)
- \(B\) = footing width

The first term in Equation 9.16 is based on the assumption that \(P_x\) acts through the centroid of the footing. The second term is based on a soil bearing pressure with an assumed eccentricity of \(B/3\), which is conservative (see Figure 5.15).
After computing $M_{cr}$, find the steel area, $A_s$, and reinforcement ratio, $p$, using Equations 9.12 and 9.15. Check if the computed $p$ is less than $p_{min}$. If so, then use $p_{min}$. Rarely will $p$ be larger than 0.0040. This light reinforcement requirement develops because we made the effective depth $d$ relatively large to avoid the need for stirrups.

TABLE 9.2 DESIGN CANTILEVER DISTANCE FOR USE IN DESIGNING REINFORCEMENT IN SPREAD FOOTINGS [15.4.2].

<table>
<thead>
<tr>
<th>Type of Column</th>
<th>$l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>$(B - c)/2$</td>
</tr>
<tr>
<td>Masonry</td>
<td>$(B - c/2)/2$</td>
</tr>
<tr>
<td>Steel</td>
<td>$(2B - (c + c_p))/4$</td>
</tr>
</tbody>
</table>

1. ACI does not specify the location of the critical section for timber columns, but in this context, it seems reasonable to treat them in the same way as concrete columns.
2. If the column has a circular, octagonal, or other similar shape, use a square with an equivalent cross-sectional area.
3. $B$ = footing width; $c$ = column width; $c_p$ = base plate width. If column has a circular or regular polygon cross section, base the analysis on an equivalent square.

Example 9.1—Part B

Using the results from Part A, design the required flexural steel.

Solution

Find the required steel area

$$l = \frac{B - c}{2} = \frac{126 - 21}{2} = 52.5 \text{ in}$$

$$M_{cr} = \frac{P_{cr}l^2}{2B} = \frac{0}{(2)(126)} = 0 \text{ in-lb}$$
Chapter 9 Spread Footings—Structural Design

9.5 Square Footings

A flexible member has a dead load moment of 200 ft·k and a live load moment of 150 ft·k. The computed nominal moment capacity, $M_n$, is 600 ft·k. Is the design of this member satisfactory? Use the ACI ultimate strength criterion.

9.2 Why are spread footings usually made of low-strength concrete?

9.3 Explain the difference between the shape of the actual shear failure surfaces in footings with those used for analysis and design.

9.4 A 400-mm square concrete column that carries a factored vertical downward load of 450 kN and a factored moment load of 100 kN·m is supported on a 1.5-m square footing. The effective depth of the concrete in this footing is 500 mm. Compute the ultimate shear force that acts on the most critical section for two-way shear failure in the footing.

9.5 A 16-in square concrete column carries vertical dead and live loads of 150 and 100 k, respectively. It is to be supported on a square footing with $f' = 3000$ lb/in² and $f = 60$ k/in². The soil has an allowable bearing pressure of 3000 lb/ft² and the groundwater table is at a great depth. Because of frost heave considerations, the bottom of this footing must be at least 30 inches below the ground surface. Determine the required footing thickness, size the flexural reinforcement, and show your design in a sketch.

9.6 A W16x50 steel column with a 22-inch square base plate is to be supported on a square spread footing. This column has a design dead load of 200 k and a design live load of 120 k. The footing will be made of concrete with $f' = 2500$ lb/in² and reinforcing steel with $f = 60$ k/in². The soil has an allowable bearing pressure of 3000 lb/ft² and the groundwater table is at a great depth. Determine the required footing thickness, size the flexural reinforcement, and show your design in a sketch.
9.8 A 500-mm square concrete column carries vertical dead and live loads of 500 and 280 kN, respectively. It is to be supported on a square footing with $f'_c = 17$ MPa and $f' = 420$ MPa. The soil has an allowable bearing pressure of 200 kPa and the groundwater table is at a great depth. Determine the required footing thickness, size the flexural reinforcement, and show your design in a sketch.

9.6 CONTINUOUS FOOTINGS

The structural design of continuous footings is very similar to that for square footings. The differences, described below, are primarily the result of the differences in geometry.

Designing for Shear

As with square footings, the depth of continuous footings is governed by shear criteria. However, we only need to check one-way shear because it is the only type that has any physical significance. The critical surfaces for evaluating one-way shear are located a distance $d$ from the face of the wall as shown in Figure 9.13.

The factored shear force acting on a unit length of the critical shear surface is:

$$ V_{m}/b = (P_{m}/b) \left( \frac{B - c - 2d}{B} \right) $$

(9.19)

Then, compute the footing thickness, $T$, using the criterion described earlier.

Designing for Flexure

Nearly all continuous footings should have longitudinal reinforcing steel (i.e., running parallel to the wall). This steel helps the footing resist flexural stresses from non-uniform loading, soft spots in the soil, or other causes. Temperature and shrinkage stresses also are a concern. Therefore, place a nominal amount of longitudinal steel in the footing (0.0018 $A_s$ to 0.0020 $A_s$) with at least two #4 bars (2 metric #13 bars). If large differential heaves or settlements are likely, we may need to use additional longitudinal reinforcement. Chapter 19 includes a discussion of this issue.

Transverse steel (that which runs perpendicular to the wall) is another issue. Most continuous footings are narrow enough so the entire base is within a 45° frustum, as shown in Figure 9.14. Thus, they do not need transverse steel. However, wider footings should include transverse steel designed to resist the flexural stresses at the critical section as defined in Table 9.2. The factored moment at this section is:

$$ M_{m}/b = (P_{m}/b) d^2 + 2 (M_{m}/b) d \left( \frac{B}{2} \right) $$

(9.21)
Factored load—Equation 2.7 governs

\[ P_{\text{fb}} = 1.4P_\text{fb} + 1.7P_\text{lb} = 1.4(120) + 1.7(88) = 318 \text{ kN/m} \]

Compute the required thickness using a shear analysis

\[ d = \frac{1500(P_{\text{fb}})(B - c)}{500 (B) \sqrt{f'_c} + 3 P_{\text{fb}}} \]

\[ = \frac{1500(318 \text{ kN/m})(1100 \text{ mm} - 200 \text{ mm})}{[500(0.85)(1100 \text{ mm}) \sqrt{15 \text{ MPa}} + (3)(318 \text{ kN/m})} \]

\[ = 237 \text{ mm} \]

For ease of construction, place the longitudinal steel below the lateral steel. Assuming metric #13 bars (diameter = 12.7 mm), the footing thickness, \( T \), is:

\[ T = d + \frac{(1/2)\text{diam. of lat. steel}}{2} + \text{diam. of long steel} + 70 \text{ mm} = 237 + 12.7/2 + 12.7 + 70 \]

\[ = 326 \text{ mm} \quad \rightarrow \quad \text{Use 400 mm} \]

In the square footing design of Example 9.1, we used an effective depth, \( d \), as the distance from the top of the footing to the contact point of the two layers of reinforcing bars (as shown in Figure 9.4). We used this definition because square footings have two-way bending, this is the average \( d \) of the two sets of rebar. However, with continuous footings we are designing only the lateral steel, so \( d \) is measured from the top of the footing to the center of the lateral bars. The longitudinal bars will be designed separately.

Design the lateral steel

\[ l = \frac{B - c/2}{2} = \frac{1.1 - 0.2/2}{2} = 0.50 \text{ m} = 500 \text{ mm} \]

\[ M_{\text{sb}} = \frac{2B}{2} \left( \frac{P_{\text{fb}}}{2} \right)^2 = 0 + (318)(0.50)^2 = 36.1 \text{ kN-m/m} \]

\[ A_{\text{sb}} = \frac{f'_{s}}{f'_{c}} \left( d - \frac{d^2}{6 f'_c} b \right) = \frac{437 \text{ mm}^2/m}{(1.176)(300 \text{ MPa})} \left( 0.311 - \frac{0.311^2}{0.9(15,000 \text{ kPa})(1 \text{ m})} \right) \]

\[ = 437 \text{ mm}^2/m \]
Check minimum steel

\[ \frac{A_s}{b} \geq 0.0020 \times (460) \times (1000) \]
\[ \geq 800 \text{ mm}^2 / \text{m} \]

Use metric #13 bars @ 150 mm OC [\( A_s = \frac{(129 \text{ mm}^2 / \text{bar}) \times (6.67 \text{ bars/m})}{6} = 880 \text{ mm}^2 / \text{m} > 800 \text{ mm}^2 / \text{m} \)]

Check development length

\[ l_d = \frac{9 \times f_y \times \sigma_y}{10 \times \sqrt{f_t} \times \left( \frac{c + K_u}{d_b} \right)} \]
\[ = 9 \times 300 \times (1)(1)(1) \times \frac{70 + 0}{12.7} \times 2.5 = 28 \]
\[ l_d = 28 \times d_b = (28)(12.7) = 355 \text{ mm} \]

\( l_d < (l_d)_{\text{required}} \), so development length is OK.

Design the longitudinal steel

\[ A_s = p b d = (0.0020) \times (1100) \times (400) = 880 \text{ mm}^2 \]

Use 7 metric #13 bars (\( A_s = 903 \text{ mm}^2 \approx 880 \text{ mm}^2 \))

The final design is shown in Figure 9.15.

**QUESTIONS AND PRACTICE PROBLEMS**

9.9 A 12-in wide concrete block wall carries vertical dead and live loads of 13.0 and 12.1 k/ft, respectively. It is to be supported on a continuous footing of 2000 lb/ft² concrete and 40 k/in² steel. The soil has an allowable bearing pressure of 4000 lb/ft², and the groundwater table is at a great depth. The local building code requires that the bottom of this footing be at least 24 inches below the ground surface. Determine the required footing thickness, and design the lateral and longitudinal steel. Show your design in a sketch.

420 MPa steel. The soil has an allowable bearing pressure of 150 kPa, and the groundwater table is at a great depth. The local building code requires that the bottom of this footing be at least 500 mm below the ground surface. Determine the required footing thickness, and design the lateral and longitudinal steel. Show your design in a sketch.

**9.7 Rectangular Footings**

Rectangular footings with width \( B \) and length \( L \) that support only one column are similar to square footings. Design them as follows:

1. Check both one-way shear (Equation 9.9) and two-way shear (Equation 9.6) using the critical shear surfaces shown in Figure 9.16a. Determine the minimum required \( d \) and \( T \) to satisfy both.
2. Design the long steel (see Figure 9.16b) by substituting \( L \) for \( B \) in Table 9.2 and Equation 9.16, and using Equation 9.17 with no modifications. Distribute this steel evenly across the footing as shown in Figure 9.16c.
3. Design the short steel (see Figure 9.16b) using Table 9.2 and Equation 9.16 with no modifications, and substituting \( L \) for \( B \) in Equation 9.17.
4. Since the central portion of the footing takes a larger portion of the short-direction flexural stresses, place more of the short steel in this zone \([15,4,4]\). To do so, divide the footing into inner and outer zones, as shown in Figure 9.16c. The portion of the total short steel area, \( A_s \), to be placed in the inner zone is \( E \).
Distribute the balance of the steel evenly across the outer zones.

**9.8 COMBINED FOOTINGS**

Combined footings are those that carry more than one column. Their loading and geometry is more complex, so it is appropriate to conduct a more rigorous structural analysis. The rigid method, described in Chapter 10, is appropriate for most combined footings. It uses a soil bearing pressure that varies linearly across the footing, thus simplifying the computations. Once the soil pressure has been established, MacGregor (1996) suggests designing the longitudinal steel using idealized beam strips ABC, as shown in Figure 9.17. Then, design the transverse steel using idealized beam strips AD. See MacGregor (1996) for a complete design example.

Large or heavily loaded combined footings may justify a beam on elastic foundation analysis, as described in Chapter 10.

**9.9 LIGHTLY-LOADED FOOTINGS**

Although the principles described in Sections 9.5 to 9.8 apply to footings of all sizes, some footings are so lightly loaded that practical minimums begin to govern the design. For example, if $P_0$ is less than about 400 kN (90 k) or $P_0/b$ is less than about 150 kN/m (10 k/ft), the minimum $d$ of 150 mm (6 in) (ACI 15.7) controls. Thus, there is no need to conduct a shear analysis, only to compute a $T$ smaller than the minimum. In the same vein, if $P_0$ is less than about 130 kN (30 k) or $P_0/b$ is less than about 60 kN/m (4 k/ft), the minimum steel requirement ($p = 0.0018$) governs, so there is no need to conduct a flexural analysis. Often, these minimums also apply to footings that support larger loads.

In addition, if the entire base of the footing is within a 45° frustum, as shown in Figure 9.14, we can safely presume that very little or no tensile stresses will develop. This is often the case with lightly loaded footings. Technically, no reinforcement is required in such cases. However, some building codes (ICBO 1806.7) have minimum reinforcement requirements for certain footings, and it is good practice to include at least the following reinforcement in all footings:

**Square Footings**
- If bottom of footing is completely within the zone of compression—no reinforcement required
- If bottom of footing extends beyond the zone of compression—as determined by a flexural analysis, but at least #4 @ 18 in o.c. each way (metric #13 @ 500 mm o.c. each way)
Connections with the Superstructure

Continuous footings
Longitudinal reinforcement
- Minimum two #4 bars (metric #13)
Lateral reinforcement
- If bottom of footing is completely within the zone of compression—no lateral reinforcement required
- If bottom of footing extends beyond the zone of compression—as determined by a flexural analysis, but at least #4 @ 18 in o.c. (metric #13 @ 500 mm o.c.)

This minimum reinforcement helps accommodate unanticipated stresses, temperature and shrinkage stresses, and other phenomena.

9.10 CONNECTIONS WITH THE SUPERSTRUCTURE

One last design feature that needs to be addressed is the connection between the footing and the superstructure. Connections are often the weak link in structures, so this portion of the design must be done carefully. A variety of connection types are available, each intended for particular construction materials and loading conditions. The design of proper connections is especially important when significant seismic or wind loads are present (Dowrick, 1987).

Connections are designed using either ASD (with the unfactored loads) or LRFD (with the factored loads) depending on the design method used in the superstructure.

Connections with Columns

Columns may be made of concrete, masonry, steel, or wood, and each has its own concerns when designing connections.

Concrete or Masonry Columns

Connect concrete or masonry columns to their footing [ACI 15.8] using dowels, as shown in Figure 9.18. These dowels are simply pieces of reinforcing bars that transmit axial, shear, and moment loads. Use at least four dowels with a total area of steel, $A_d$, at least equal to that of the column steel or 0.005 times the cross-sectional area of the column, whichever is greater. They may not be larger than #11 bars [ACI 15.8.2.3] and must have a 90° hook at the bottom. Normally, the number of dowels is equal to the number of vertical bars in the column.

Design for Compressive Loads

Check the bearing strength of the footing [ACI 10.17] to verify that it is able to support the axial column load. This is especially likely to be a concern if the column carries large compressive stresses that might cause something comparable to a bearing capacity failure.
inside the footing. To check this possibility, compute the factored column load, \( P_n \), and compare it to the nominal column bearing capacity, \( P_{n0} \):

\[
P_{n0} = 0.85 f'_{c} A_{1} s
\]  
\[ (9.23) \]

Then, determine whether the following statement is true:

\[
P_n \leq \phi P_{n0}
\]  
\[ (9.24) \]

Where:
- \( P_n \) = factored column load
- \( P_{n0} \) = nominal column bearing capacity (i.e., bearing of column on top of footing)
- \( f'_{c} \) = 28-day compressive strength of concrete
- \( s = (A_1/A_{2})^{0.5} \leq 2 \) if the frustum in Figure 9.19 fits entirely within the footing (i.e., if \( c + 4d \leq B \))
- \( s = 1 \) if the frustum in Figure 9.19 does not fit entirely within the footing
- \( A_1 = \) cross-sectional area of the column = \( c_1 \)
- \( A_{2} = (c + 4d)^2 \) as shown in Figure 9.19
- \( c = \) column width or diameter
- \( \phi = \) resistance factor = 0.7 [ACI 9.3.2.4]

If Equation 9.24 is not satisfied, use a higher strength concrete (greater \( f'_{c} \)) in the footing or design the dowels as compression steel.

**Design for Moment Loads**

If the column imparts moment loads onto the footing, then some of the dowels will be in tension. Therefore, the dowels must be embedded at least one development length into the footing, as shown in Figure 9.20 and defined by the following equations [ACI 12.5]:

\[
l_{d} = \frac{1200 d}{f'_{c}}
\]  
\[ (9.25 \text{ English}) \]
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Figure 9.20  Minimum required embedment of dowels subjected to tension.

9.10 Connections with the Superstructure

As long as the number and size of dowels are at least as large as the vertical steel in the column, then they will have sufficient capacity to carry the moment loads.

Design for Shear Loads

If the column also imparts a shear load, $V_v$, onto the footing, the connection must be able to transmit this load. Since the footing and column are poured separately, there is a weak shear plane along the cold joint. Therefore, the dowels must transmit all of the applied shear load. The minimum required dowel steel area is:

\[
A_d = \frac{V_v}{\phi f_{y} \mu}
\]

Where:
- $A_d$ = minimum required dowel steel area
- $V_v$ = applied factored shear load
- $\phi = 0.85$ for shear
- $f_{y}$ = yield strength of reinforcing steel
- $\mu = 0.6$ if the cold joint not intentionally roughened or 1.0 if the cold joint is roughened by heavy raking or grooving [ACI 11.7.4.3]

However, the ultimate shear load, $V_u$, cannot exceed $0.2 f_c' A_c$, where $f_c'$ is the compressive strength of the column concrete, and $A_c$ is the cross-sectional area of the column.

Splices

Most designs use a lap splice to connect the dowels and the vertical column steel. However, some columns have failed in the vicinity of these splices during earthquakes, as shown in Figure 9.21. Therefore, current codes require much more spiral reinforcement in columns subjected to seismic loads. In addition, some structures with large moment loads, such as certain highway bridges, require mechanical splices or welded splices to connect the dowels and the column steel.

Steel Columns

Steel columns are connected to their foundations using base plates and anchor bolts, as shown in Figure 9.22. The base plates are welded to the bottom of the columns when they are fabricated, and the anchor bolts are cast into the foundation when the concrete is placed. The column is then erected over the foundation, and the anchor bolts are fit through predrilled holes in the base plate.

The top of the footing is very rough and not necessarily level, so the contractor must use special construction methods to provide uniform support for the base plate and to make the column plumb. For traffic signal poles, light standards, and other lightweight columns, the most common method is to provide a nut above and below the base plate, as

\[
l_d = \frac{100 d}{\sqrt{f_c'}}
\]

(9.25 SI)

Where:
- $T$ = footing thickness (in, mm)
- $l_d$ = development length for 90° hooks, as defined in Figure 9.20 (in, mm)
- $d$ = bar diameter (in, mm)
- $f_c'$ = 28-day compressive strength of concrete (lb/in², MPa)

The development length computed from Equation 9.25 may be modified by the following factors [ACI 12.5.3]²:

- For standard reinforcing bars with yield strength other than 60,000 lb/in²: $f_y/60,000$
- For metric reinforcing bars with yield strength other than 420 lb/in²: $f_y/420$
- If at least 50 mm (2 in) of cover is present beyond the end of the hook: 0.7

Sometimes this development length requirement will dictate a footing thickness $T$ greater than that required for shear (as computed earlier in this chapter).

²This list only includes modification factors that are applicable to anchorage of vertical steel in retaining wall footings. ACI 12.5.3 includes additional modification factors that apply to other situations.
Connections with the Superstructure

Figure 9.21 Imperial County Services Building, El Centro, California. The bases of these columns failed during the 1979 El Centro earthquake, causing the building to sag about 300 mm. As a result, this six-story building had to be demolished. (U.S. Geological Survey photo)

Figure 9.22 Base plate and anchor bolts to connect a steel column to its foundation.

Figure 9.23 Methods of leveling the base plate: a) Double nuts, b) Blocks and shims.

shown in Figure 9.23a, and adjust these nuts as needed to make the column plumb. However, columns for buildings, bridges, and other large structures are generally too heavy for this method, so the contractor must temporarily support the base plate on steel blocks and shims, and clamp it down with a single nut on each anchor bolt, as shown in Figure 9.23b. These shims are carefully selected to produce a level base plate and a plumb column. Other construction methods also have been used.

Once the column is securely in place and the various members that frame into it have been erected, the contractor places a nonshrink grout between the base plate and the footing. This grout swells slightly when it cures (as compared to normal grout, which shrinks), thus maintaining continuous support for the base plate. The structural loads from the column are then transmitted to the footing as follows:

- Compressive loads are spread over the base plate and transmitted through the grout to the footing.
- Tensile loads pass through the base plate and are resisted by the anchor bolts.
- Moment loads are resisted by a combination of compression through the grout and tension in half of the bolts.
- Shear loads are transmitted through the anchor bolts, through sliding friction along the bottom of the base plate, or possibly both.

Design Principles

The base plate must be large enough to avoid exceeding the nominal bearing strength of the concrete (see earlier discussion under concrete and masonry columns). In addition, it must be thick enough to transmit the load from the column to the footing. The design of base plates is beyond the scope of this book, but it is covered in most steel design texts and in DeWolf and Ricker (1990).
Anchor bolts can fail either by fracture of the bolts themselves, or by loss of anchorage in the concrete. Steel is much more ductile than the concrete, and this ductility is important, especially when wind or seismic loads are present. Therefore, anchor bolts should be designed so the critical mode of failure is shear or tension of the bolt itself rather than failure of the anchorage. In other words, the bolt should fail before the concrete fails.

The following methods may be used to design anchor bolts that satisfy this principle. These methods are based on ACI and AISC requirements, but some building codes may impose additional requirements, or specify different design techniques, so the engineer must check the applicable code.

**Selection and Sizing of Anchor Bolts**

Five types of anchor bolts are available, as shown in Figure 9.24:

- **Proprietary anchor bolts** are patented designs that often are intended for special applications, principally with wood-frame structures.

- **Drilled-in anchor bolts** are used when a cast-in-place anchor bolt was not installed during construction of the footing. They are constructed by drilling a hole in the concrete, then embedding a threaded rod into the hole and anchoring it using either epoxy grout or mechanical wedges. This is the most expensive of the five types and is usually required only to rectify mistakes in the placement of conventional anchor bolts.

- **Standard structural steel bolts** may simply be embedded into the concrete to form anchor bolts. These bolts are similar to those used in bolted steel connections, except they are much longer. Unfortunately, these bolts may not be easily available in lengths greater than about 6 inches, so they often are not a practical choice.

- **Structural steel rods that have been cut to length and threaded** form anchor bolts that are nearly identical to a standard steel bolts and have the advantage of being more readily available. This is the most common type of anchor bolt for steel columns. If one nut is used at the bottom of each rod, it should be tack welded to prevent the rod from turning when the top nut is tightened. Alternatively, two nuts may be used.

- **Hooked bars** (also known as an L-bolts or a J-bolts) are specially fabricated fasteners made for this purpose. These are principally used for wood-frame structures, and are generally suitable for steel structures only when no tensile or shear loads are present.

In addition, the design must satisfy AISC requirements for interaction between shear and tensile stresses. Figure 9.25 presents the shear and tensile capacities for ASTM A36 and ASTM A307 bolts that satisfies Equations 9.27 and 9.28 and the interaction requirements, and may be used to select the required diameter.

Typically four anchor bolts are used for each column. It is best to place them in a square pattern to simplify construction and leave less opportunity for mistakes. Rectangular or hexagonal patterns are more likely to be accidentally built with the wrong orientation. More bolts and other patterns also may be used, if necessary.

If the design loads between the column and the footing consist solely of compression, then anchor bolts are required only to resist erection loads, accidental collisions during erection, and unanticipated shear or tensile loads. The engineer might attempt to estimate these loads and design accordingly, or simply select the bolts using engineering judgement. Often these columns simply use the same anchor bolt design as nearby columns, thus reducing the potential for mistakes during construction.

**Anchorage**

Once the bolt diameter has been selected, the engineer must determine the required depth of embedment into the concrete to provide the necessary anchorage. The required embedment depends on the type of anchor, the spacing between the anchors, the kind of loading.
Some engineers rely on both sources of shear resistance, while others rely only on one or the other.

When transferring shear loads through the anchor bolts, the engineer must recognize that the bolt holes in the base plate are oversized in order to simplify the erection of the column onto the footing. As a result, the resulting gap between the bolts and the base plate does not allow for efficient transfer of the shear loads. The base plate may need to move laterally before touching the bolts, and most likely only some of the bolts will become fully engaged with the plate. Therefore, when using this mode of shear transfer, it is probably prudent to assume the shear load is carried by only half of the bolts.

So long as the grout has been carefully installed between the base plate and the footing, and the base plate has not been installed using the double nut method as shown in Figure 9.23a, there will be sliding friction along the bottom of the base plate. AISC recommends using a coefficient of friction of 0.55 for conventional base plates, such as that shown in Figure 9.23b, and the resistance factor, \( \phi \), is 0.90. Thus, the available sliding friction resistance, \( \phi V'' \), is:

\[
\phi V'' = \phi \mu P
\]

\[
= (0.90)(0.55) P
\]

\[
= 0.50 P
\]  

(9.29)

The value of \( P \) in Equation 9.29 should be the lowest unfactored normal load obtained from Equations 2.1 to 2.4. Usually Equation 2.1 governs, except when uplift wind or seismic loads are present, as described in Equation 2.4. It is good practice to ignore any normal stress produced by live loads or the clamping forces from the nuts.

Sometimes short vertical fins are welded to the bottom of the base plate to improve shear transfer to the grout. These fins may just raise the coefficient of friction to 0.7.

**Example 9.3**

A steel wide flange column with a steel base plate is to be supported on a spread footing. The AISC factored design loads are: \( P = 270 \text{ k} \) compression and \( M = 200 \text{ ft-k} \). Design an anchor bolt system for this column using four bolts arranged in a 15\( \times \)15 inch square.

**Solution**

Reduce the applied loads to a couple separated by 15 in:

\[
P = \frac{270 \text{ k}}{2} \pm \frac{200 \text{ ft-k}}{15/12} \text{ ft}
\]

\[
= 135 \pm 160 \text{ k}
\]

There are two bolts on each side, so the maximum tensile force in each bolt is:
The shear force is zero.
Per Figure 9.25, use 3/4 inch bolts.
The depth of embedment should be \((12)(0.75) = 9\) in.
Use four 3/4 inch diameter x 13 inch long A36 threaded rods embedded 9 inches into the footing. Firmly tighten two nuts at the bottom of each rod.

**Wood Columns**

Wood columns, often called *posts*, usually carry light loads and require relatively simple connections. The most common type is a metal bracket, as shown in Figure 9.26. These are set in the wet concrete immediately after it is poured. The manufacturers determine the allowable loads and tabulate them in their catalogs (for example, see www.strongtie.com).

It is poor practice to simply embed a wooden post into the footing. Although at first this would be a very strong connection, in time the wood will rot and become weakened.

![Figure 9.26 Steel post base for securing a wood post to a spread footing (Simpson Strong-Tie Co., Inc.)](image)

**Connections with the Superstructure**

The connection between a concrete or masonry wall and its footing is a simple one. Simply extend the vertical wall steel into the footing [ACI 15.8.2.2], as shown in Figure 9.27. For construction convenience, design this steel with a lap joint immediately above the footing.

The design of vertical steel in concrete retaining walls is discussed in Chapter 24. Design procedures for other walls are beyond the scope of this book.

Connect wood-frame walls to the footing using anchor bolts, as shown in Figure 9.28. Normally, standard building code criteria govern the size and spacing of these bolts. For example, the *Uniform Building Code* (ICBO, 1997) specifies 1/2 in (12 mm) nominal diameter bolts embedded at least 7 in (175 mm) into the concrete. It also specifies bolt spacings of no more than 6 ft (1.8 m) on center.

Sometimes we must supply a higher capacity connection between wood frame walls and footings, especially when large uplift loads are anticipated. Steel holdown brackets, such as that shown in Figure 9.29, are useful for these situations.

Many older wood-frame buildings have inadequate connections between the structure and its foundation. Figure 9.30 shows one such structure that literally fell off its foundation during the 1989 Loma Prieta Earthquake in Northern California.

Some wood-frame buildings have failed during earthquakes because the *cripple walls* collapsed. These are short wood-frame walls that connect the foundation to the floor. They may be retrofitted by installing plywood shear panels or by using diagonal steel bracing (Shepherd and Delos-Santos, 1991).

![Figure 9.27 Connection between a concrete or masonry wall and its footing.](image)
Chapter 9 Spread Footings—Structural Design

9.10 Connections with the Superstructure

Questions and Practice Problems

9.11 An 18-inch square concrete column carries dead and live compressive loads of 240 and 220 k, respectively. It is to be supported on a 8 ft wide and 12 ft long rectangular spread footing. Select appropriate values for $f'$ and $f$, then determine the required footing thickness and design the flexural reinforcing steel. Show the results of your design in a sketch.

9.12 The column described in Problem 9.11 is reinforced with 6 #8 bars. Design the dowels required to connect it with the footing, and show your design in a sketch.

9.13 A 400-mm diameter concrete column carrying a factored compressive load of 1500 kN is supported on a 600-mm thick, 2500-mm-wide square spread footing. It is reinforced with eight metric #19 bars. Using $f'_c = 18$ MPa and $f = 420$ MPa, design the dowels for this connection.

9.14 A 24-inch square concrete column carries a factored compressive load of 900 k and a factored shear load of 100 k. It is to be supported on a spread footing with $f'_c = 3000$ lb/ft$^2$ and $f = 60$ k/lin. The column is reinforced with twelve #9 bars. Design the dowels for this connection.

9.15 A steel column with a square base plate is to be supported on a spread footing. The AISC factored design loads are: $P' = 600$ k compression and $V' = 105$ k. Design an anchor bolt system for this base plate and show your design in a sketch.
Chapter 9  Spread Footings—Structural Design

Summary

Major Points

1. The plan dimensions and minimum embedment depth of a spread footing are governed by geotechnical concerns, and are determined using the unfactored loads.

2. The thickness and reinforcement of a spread footing are governed by structural concerns. Structural design is governed by the ACI code, which means these analyses are based on the factored load.

3. The structural design of spread footings must consider both shear and flexural failure modes. A shear failure consists of the column or wall punching through the footing, while a flexural failure occurs when the footing has insufficient cantilever strength.

4. Since we do not wish to use stirrups (shear reinforcement), we conduct the shear analysis first and select an effective depth, \( d \), so the footing that provides enough shear resistance in the concrete to resist the shear force induced by the applied load. This analysis ignores the shear strength of the flexural steel.

5. Once the shear analysis is completed, we conduct a flexural analysis to determine the amount of steel required to provide the needed flexural strength. Since \( d \) is large, the required steel area will be small, and it is often governed by \( P_{min} \).

6. For square footings, use the same flexural steel in both directions. Thus, the footing is reinforced twice.

7. For continuous footings, the lateral steel, if needed, is based on a flexural analysis. Use nominal longitudinal steel to resist nonuniformities in the load and to accommodate inconsistencies in the soil bearing pressure.

8. Design rectangular footings similar to square footings, but group a greater portion of the short steel near the center.

9. Practical minimum dimensions will often govern the design of lightly loaded footings.

10. The connection between the footing and the superstructure is very important. Use dowels to connect concrete or masonry structures. For steel columns and wood-frame walls, use anchor bolts. For wood posts, use specially manufactured brackets.

Vocabulary

Anchor bolts  Effective depth  Reinforcing bars
Critical section for bending  Factored load  Shear failure
Critical shear surface  Flexural failure  Two-way shear
Development length  Minimum steel  Unfactored load
Dowels  One-way shear  28-day compressive strength

Comprehensive Questions and Practice Problems

9.16 A 400-mm square concrete column reinforced with eight metric #19 bars carries vertical dead and live loads of 980 and 825 kN, respectively. It is to be supported on a 2.0 m x 3.5 m rectangular footing. The concrete in the footing will have \( f'_c = 20 \) MPa and \( f'_c = 400 \) MPa. The building will have a slab-on-grade floor, so the top of the footing must be at least 150 mm below the finish floor elevation. Develop a complete structural design, including dowels, and show it in a sketch.

9.17 A 12-in wide masonry wall carries dead and live loads of 3 k/ft and 8 k/ft, respectively and is reinforced with #6 bars at 24 inches on center. This wall is to be supported on a continuous footing with \( f'_c = 2000 \) lb/in\(^2\) and \( f'_c = 60 \) k/ft\(^2\). The underlying soil has an allowable bearing pressure of 3000 lb/ft\(^2\). Develop a complete structural design for this footing, including dowels, and show your design in a sketch.