

## **STRESSES IN SOIL**

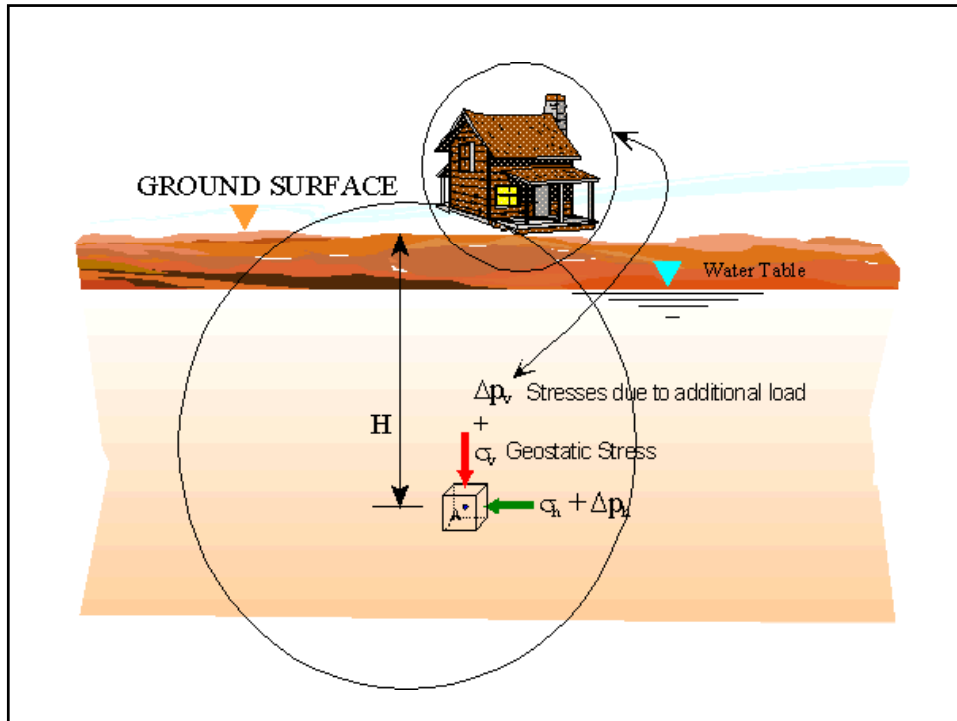
Stresses at a point in a soil layer are caused by:

- 1- **Self weight of the soil** layers (Geostatic Stresses)
- 2- **Added load** (Such as buildings, bridges, dams, etc.)

*Stresses at a point in a soil mass are divided into two main types:*

*I- **Geostatic Stresses** ----- Due to the self weight of the soil mass.*

*II- **Excess Stresses** ----- From structures*



## Distributed Loads

- **Strip Loads ( $L/B > 9$ )**
  - Wall Footings
  - Embankments
- **Circular Loads (R)**
  - Storage Tanks
- **Rectangular Loads ( $B \times L$ )**
  - Spread Footings
  - Mat Foundations

## **I. Geostatic stresses**

### **I.A. Vertical Stress**

*Vertical geostatic stresses increase with depth,*

*There are 3 types of geostatic stresses:*

- a. Total Stress,  $\sigma_{total}$*
- b. Effective Stress,  $\sigma'$*
- c. Pore Water Pressure,  $u$*

*Total Stress = Effective stress + Pore Water Pressure*

$$\sigma_{total} = \sigma' + u$$

### **I.B. Horizontal Stress or Lateral Stress**

$$\sigma_h = K_o \sigma'_v$$

$K_o$  = Lateral Earth Pressure Coefficient

- *For normally consolidated soils:*

$$K_o = 1 - \sin \phi'$$

- *For over consolidated soils:*

$$K_o = (1 - \sin \phi')(\text{OCR})^{\sin \phi'}$$

## ***II. Stress Distribution in Soil Mass:***

***When applying a load on a half space medium the excess stresses in the soil will decrease with depth.***

***Like in the geostatic stresses, there are vertical and lateral excess stresses.***

### ***1. Vertical Stress Due to a Point Load***

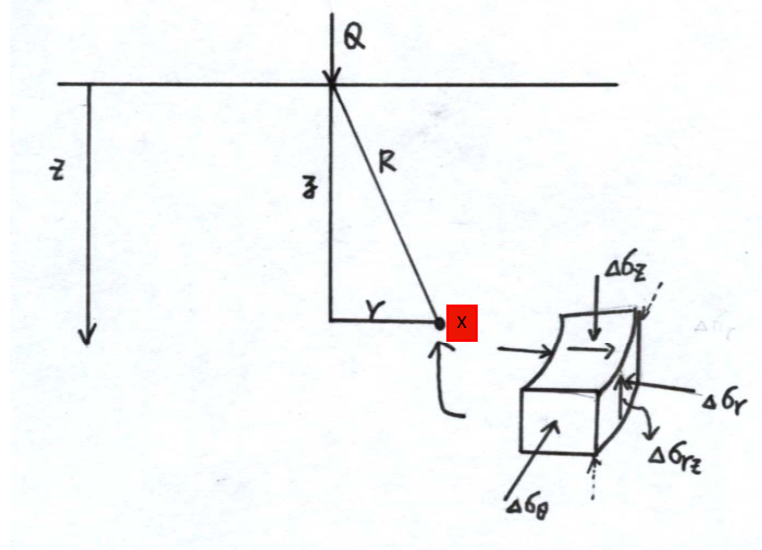
Boussinesq's solution for elastic behaviour

Boussinesq (1885) solved the problem of stress distribution at any point ( $X$ ) in a:

**semi-infinite half space** (infinite in depth) of  
**Homogenous** (same soil properties with depth),  
**isotropic** (same soil properties in all directions) and  
**elastic** (fully recoverably strains) material

as a result of a point load ( $Q$ ) applied on the surface:

## Point Load Stresses



$$\Delta\sigma_z = \frac{3Q}{2\pi z^2} \left( \frac{1}{1 + (r/z)^2} \right)^{5/2} = \frac{3Qz^3}{2\pi R^5} = \frac{Q}{z^2} I$$

where **I** is an influence factor, and

$$I = \frac{3}{2\pi} \left( \frac{1}{1 + (r/z)^2} \right)^{5/2}$$

$$\Delta\sigma_\theta = -\frac{Q}{2\pi} (1-2\nu) \left( \frac{z}{(r^2 + z^2)^{3/2}} - \frac{1}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right)$$

$$\Delta\sigma_r = \frac{Q}{2\pi} \left( \frac{3r^2 z}{(r^2 + z^2)^{5/2}} - \frac{1-2\nu}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right)$$

$\nu$  is the Poisson's ratio

## Boussinesq's solution

where

$Q$  = surface point load

$z$  = depth of the point  $X$  below  $Q$

$r$  = the horizontal distance of point  $X$  from  $Q$

$I_p$  = point load influence factor for vertical stress change

(available in standard tables or charts)

$I_p$  = Influence factor for the point load

Knowing  $r/z$  -----  $I_p$  can be obtained from tables

Influence factors for vertical stress increase due to a point load (Craig, 1997).

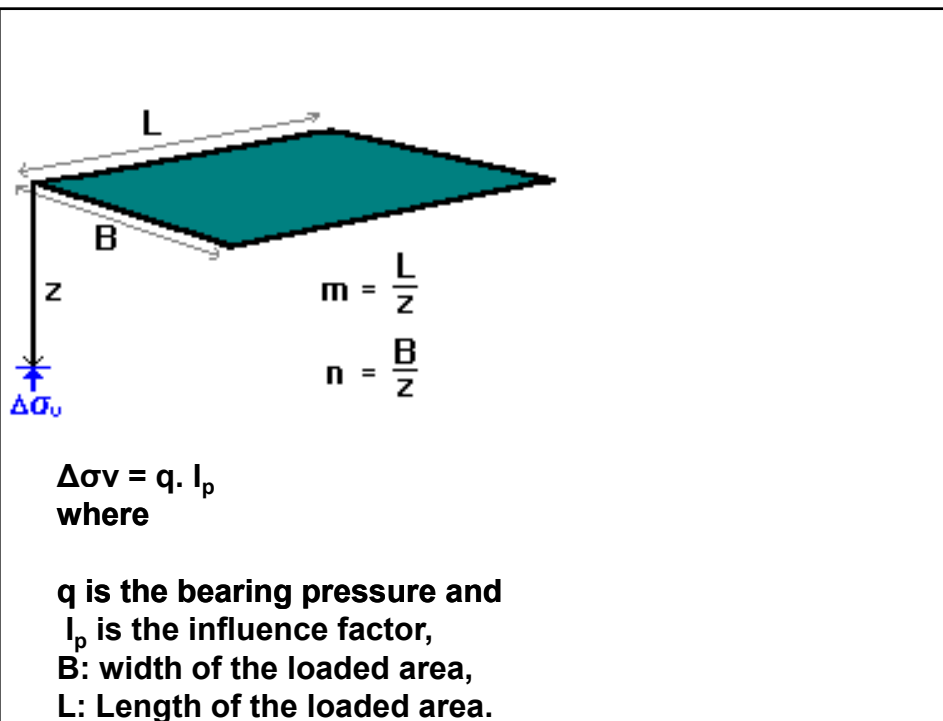
$r/z$	$I$	$r/z$	$I$	$r/z$	$I$
0.0	0.478	0.8	0.139	1.6	0.020
0.1	0.466	0.9	0.108	1.7	0.016
0.2	0.433	1.0	0.084	1.8	0.013
0.3	0.385	1.1	0.066	1.9	0.011
0.4	0.329	1.2	0.051	2.0	0.009
0.5	0.273	1.3	0.040	2.2	0.006
0.6	0.221	1.4	0.032	2.4	0.004
0.7	0.176	1.5	0.025	2.6	0.003

## 2. Vertical Stress Under Corner of a Rectangular Area Carrying Uniform Pressure

The vertical stress at a depth  $z$  below the corner of a rectangular area subject to uniform pressure is

$$\Delta\sigma_z \text{ (or } \Delta\sigma_v) = q \cdot I_R \text{ or } q \cdot I_p$$

$$I_R = \frac{1}{4\pi} \left[ \frac{2mn\sqrt{m^2+n^2+1} \left( \frac{m^2+n^2+2}{m^2+n^2+1} \right)}{m^2+n^2+m^2n^2+1} + \tan^{-1} \left( \frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1} \right) \right]$$



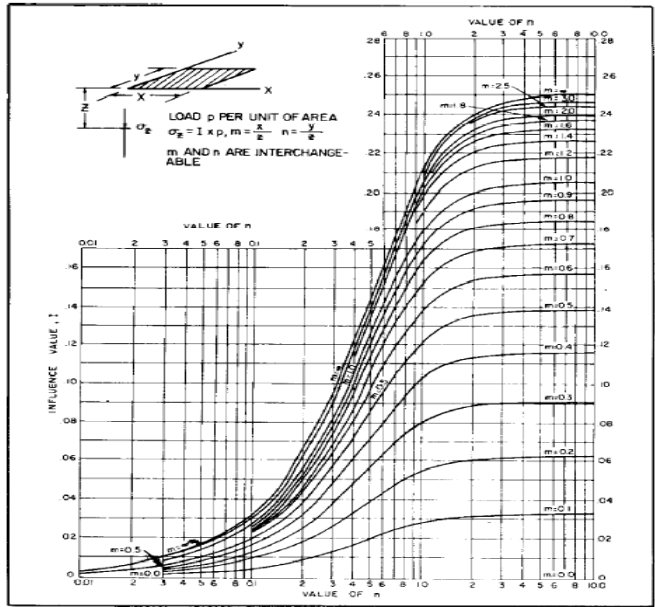


FIGURE 4  
 Influence Value for Vertical Stress Beneath a Corner of a Uniformly Loaded Rectangular Area (Boussinesq Case)

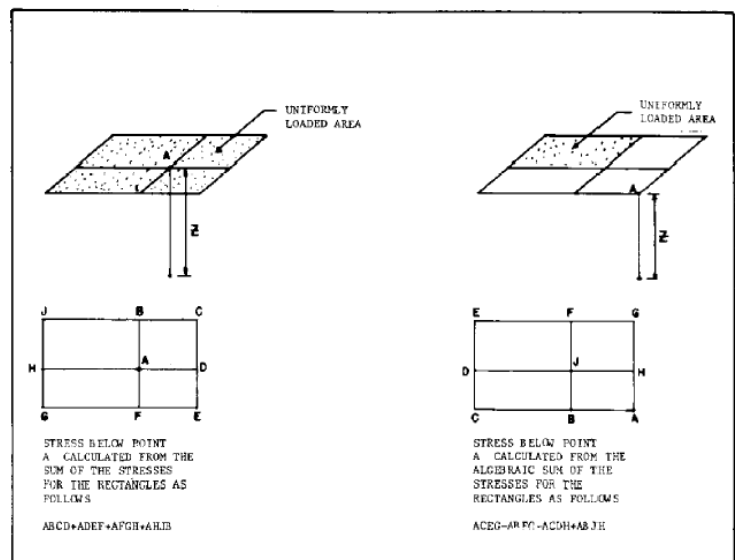


FIGURE 9  
 Determination of Stress Below Corner of Uniformly Loaded Rectangular Area



## 4. For a Circular Loaded Area

The excess vertical stress (beneath centre)

$$\Delta p = q \left[ 1 - \frac{1}{[(R/z)^2 + 1]^{3/2}} \right]$$

- $q$  is the uniformly distributed pressure on the circular area

$$\Delta p \text{ or } \Delta \sigma_z = q \cdot I$$

$q$  = surface contact pressure

$z$  = depth

$r$  = radius of uniformly loaded area.

$x$  = horizontal distance from the center of the circular area.

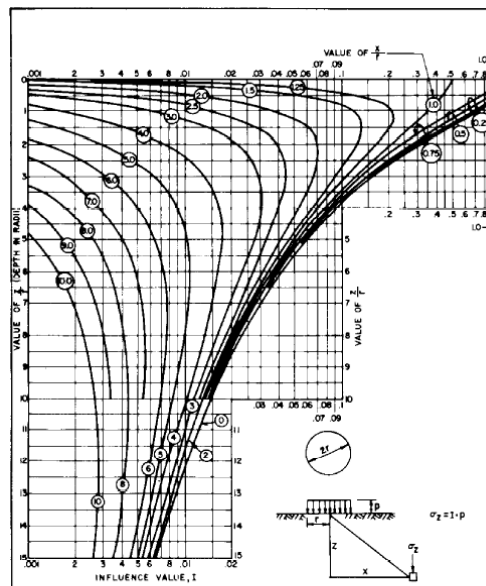


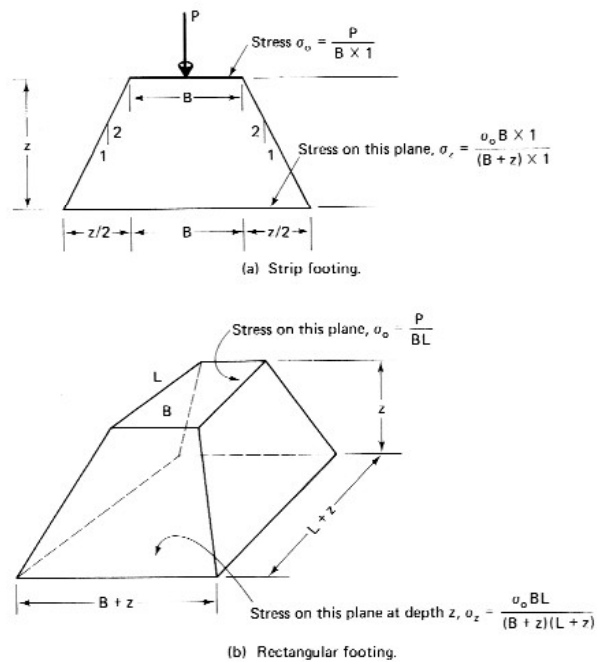
FIGURE 5  
Influence Value for Vertical Stress Under Uniformly Loaded Circular Area  
(Roussinesq Case)

## 5. The 2:1-Method

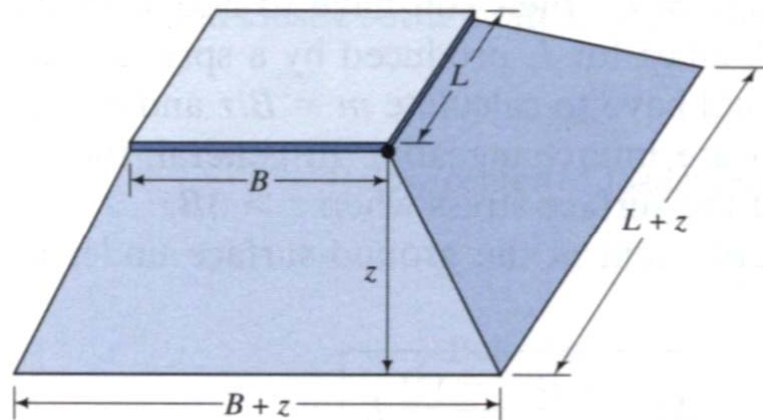
An approximate, but very simple way of looking at the vertical stress distribution with depth.

It assumes that the influence of the load area spreads at 2:1 (1 horizontal to 2 vertical) and

that the same pressure is then distributed over the larger area - the so called 2:1 method.



The approximate method is reasonably accurate (compared with Boussinesq's elastic solution) when  $z > B$ .



### Approximate method for rectangular loads

In preliminary analyses of vertical stress increase under the center of rectangular loads, geotechnical engineers often use an approximate method (sometimes called the 2:1 method).

The vertical stress increase under the center of the load is

$$\Delta\sigma_z = \frac{q_s BL}{(B + z)(L + z)}$$